

## 1-Introduction

### 1-1 Introduction to Communication Engineering :

The principle objective of a communication system is to transmit information signals from one point to another .

The information signals may be the result of a voice message , a T.V. picture , a meter reading , or may take on a variety of other formats depending on the specific application . Communication Engineering involves the analysis , design , and fabrication of an operating system that performs the communication objective .

### 1-2 Functional Elements of a Communication System :

Fig 1.1 shows a commonly used model for a single- link communication system . Although it suggests a system for communication between two remotely located points , this block diagram is also applicable to remote sensing systems , such as radar or sonar , in which the system input and output may be located at the same site .

Regardless of the particular application and configuration , all information transmission systems involve three major subsystems :

-a transmitter , the channel , and a receiver .

We will now discuss briefly each functional element shown in Fig. 1-1 .

#### Input Transducer:

The wide variety of possible sources of information results in many different forms for messages . Messages may be analog or digital . The message produced by a source must be converted by a transducer to a form suitable for the particular type of communication system employed .For example, in electrical communications, speech waves are converted by a microphone to voltage variations .Such a converted message is referred to as the message signal .

#### Transmitter:

The purpose of the transmitter is to couple the message to the channel .

It is often necessary to modulate a carrier wave with the signal from the input transducer .

Modulation is the systematic variation of some attribute of the carrier, such as amplitude , phase , or frequency , in accordance with a function of the message signal .

There are several reasons for using a carrier and modulating it . Important ones are :

(1) for ease of radiation , (2) to reduce noise and interference , (3) for channel assignment , (4) for multiplexing or transmission of several messages over a single channel , and (5) to overcome equipment limitations .

## Channel :

The channel can have many different forms ; the most familiar is the channel that exists between the transmitting antenna of a commercial radio station and the receiving antenna of a radio . In this channel , the transmitted signal propagates through the atmosphere , or free space , to the receiving antenna .

Other forms of channels are :

- Transmission lines (such as open two- wire systems and co-axial cables) .
- Optical fiber channels .
- Guided electromagnetic – wave channels .

All channels have one thing in common : the signal undergoes degradation from transmitter to receiver . This degradation results from noise and other undesired signals or interference but also may include other distortion effects as well, such as fading signal levels, multiple transmission paths, and filtering .

## Receiver:

The receiver's function is to extract the desired message from the received signal at the channel output and to convert it to a form suitable for the output transducer . Although amplification may be one of the first operations performed by the receiver , where the received signal may be extremely weak , the main function of the receiver is to demodulate the received signal .

Often it is desired that the receiver output be a scaled , possibly delayed , version of the message signal at the modulator input .

## Output Transducer:

The output transducer completes the communication system . The device converts the electric signal at its input into the form desired by the system user. The most common output transducer is a loudspeaker . There are many other examples , such as tape recorders, personal computers, meters , and cathode – ray tubes.

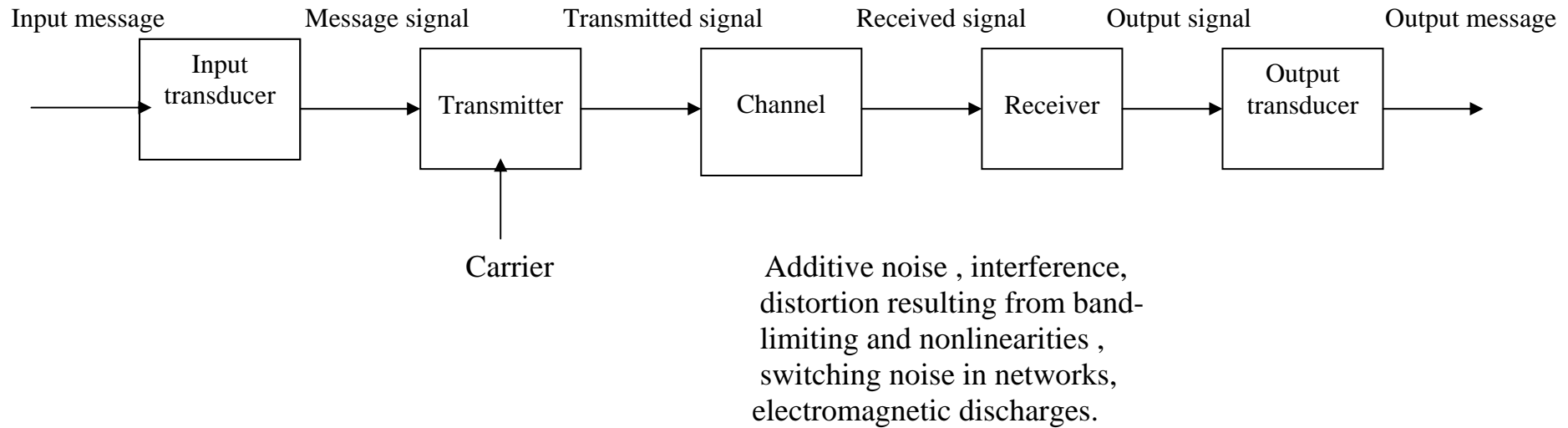


Fig. 1-1 The Block Diagram of a Communication System .

## 2. Signals and Systems

### 2-1 Introduction :

Signals are time-varying quantities such as voltages or current .

A system is a combination of devices and networks (subsystems) chosen to perform a desired function .Because of the sophistication of modern communication systems , a great deal of analysis and experimentation with trial subsystems occurs before actual building of the desired system . Thus the communications engineer's tools are mathematical models for signals and systems .

### 2-2 Classification of Signals:

#### 2-2-1 Continuous-time and discrete-time signals

By the term continuous-time signal we mean a real or complex function of time  $s(t)$ , where the independent variable  $t$  is continuous.

If  $t$  is a discrete variable, i.e.,  $s(t)$  is defined at discrete times, then the signal  $s(t)$  is a discrete-time signal. A discrete-time signal is often identified as a sequence of numbers ,denoted by  $\{s(n)\}$ , where  $n$  is an integer.

#### 2-2-2 Analogue and digital signals :

If a continuous-time signal  $s(t)$  can take on any values in a continuous time interval, then  $s(t)$  is called an analogue signal.

If a discrete-time signal can take on only a finite number of distinct values,  $\{s(n)\}$ , then the signal is called a digital signal.

#### 2-2-3 Deterministic and random signals :

Deterministic signals are those signals whose values are completely specified for any given time.

Random signals are those signals that take random values at any given times.

#### 2-2-4 Periodic and nonperiodic signals :

A signal  $s(t)$  is a periodic signal if  $s(t) = s(t + nT_0)$ , where  $T_0$  is called the period and the integer  $n > 0$ .

If  $s(t) \neq s(t + T_0)$  for all  $t$  and any  $T_0$ , then  $s(t)$  is a nonperiodic or aperiodic signal.

#### 2-2-5 Power and energy signals :

A complex signal  $s(t)$  is a power signal if the average normalized power  $P$  is finite, where

$$0 < P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)s^*(t)dt < \infty$$

and  $s^*(t)$  is the complex conjugate of  $s(t)$ .

A complex signal  $s(t)$  is an energy signal if the normalized energy  $E$  is finite, where

$$0 < E = \int_{-\infty}^{\infty} s(t)s^*(t) dt = \int_{-\infty}^{\infty} |s(t)|^2 dt < \infty$$

In communication systems, the received waveform is usually categorized into the desired part, containing the information signal, and the undesired part, called noise.

## 2-3 Some Useful Functions :

### a: Unit impulse function :

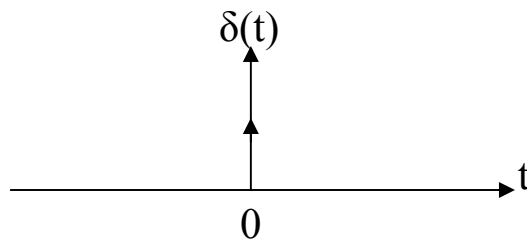
The unit impulse function, also known as the Dirac delta function,  $\delta(t)$ , is defined by :

$$\int_{-\infty}^{\infty} s(t)\delta(t)dt = s(0)$$

An alternative definition is :

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \dots \dots \text{and}$$

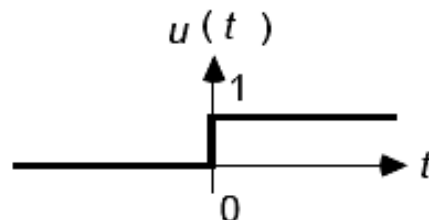
$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



### b: Unit step function :

The unit step function  $u(t)$  is :

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



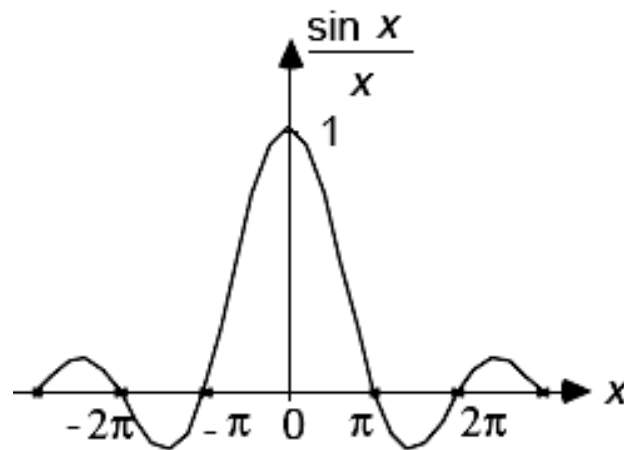
and the unit step function is related to the unit impulse function by :

$$u(t) = \int_{-\infty}^{\infty} \delta(t) dt \quad \text{and} \quad \frac{du(t)}{dt} = \delta(t)$$

c: Sampling function :

A sampling function is denoted by :

$$\text{Sa}(x) = \frac{\sin x}{x}$$



d: Sinc function :

A sinc function is denoted by :

$$\text{sinc } x = \frac{\sin \pi x}{\pi x}$$

Hence,

$$\text{Sa}(x) = \text{sinc} \left( \frac{x}{\pi} \right)$$

e: Rectangular function :

A single rectangular pulse is denoted by :

$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \leq \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases}$$

f: Triangular function :

$$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases}$$

2-4 Other Useful Operations :

a; Cross-correlation :

The cross-correlation of two real-valued power waveforms  $s_1(t)$  and  $s_2(t)$  is defined by :

$$R_{12}(\tau) = \langle s_1(t) s_2(t + \tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} s_1(t) s_2(t + \tau) dt$$

If  $s_1(t)$  and  $s_2(t)$  are periodic with the same period  $T_0$ , then

$$R_{12}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s_1(t) s_2(t + \tau) dt$$

The cross-correlation of two real-valued energy waveforms  $s_1(t)$  and  $s_2(t)$  is defined by :

$$R_{12}(\tau) = \int_{-\infty}^{\infty} s_1(t) s_2(t + \tau) dt$$



Correlation is a useful operation to measure the similarity between two waveforms. To compute the correlation between waveforms, it is necessary to specify which waveform is being shifted. In general,  $R_{12}(\tau)$  is not equal to  $R_{21}(\tau)$ , where

$$R_{21}(\tau) = \langle s_2(t) s_1(t + \tau) \rangle.$$

The cross-correlation of two complex waveforms is :

$$R_{12}(\tau) = \langle s_1^*(t) s_2(t + \tau) \rangle.$$

b: Auto-correlation :

The auto-correlation of a real-valued power waveform  $s_1(t)$  is defined by :

$$R_{11}(\tau) = \langle s_1(t) s_1(t + \tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} s_1(t) s_1(t + \tau) dt$$

If  $s_1(t)$  is periodic with fundamental period  $T_0$ , then

$$R_{11}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s_1(t) s_1(t + \tau) dt$$

The auto-correlation of a real-valued energy waveform  $s_1(t)$  is defined by :

$$R_{11}(\tau) = \int_{-\infty}^{\infty} s_1(t) s_1(t + \tau) dt$$

The auto-correlation of a complex power waveform is :

$$R_{11}(\tau) = \langle s_1^*(t) s_1(t + \tau) \rangle.$$

c: Convolution :

The convolution of a waveform  $s_1(t)$  with a waveform  $s_2(t)$  is given by

$$s_3(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\lambda) s_2(t-\lambda) d\lambda$$

$$s_3(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\lambda) s_2[-(\lambda-t)] d\lambda$$

where \* denotes the convolution operation. The above equation is obtained by :

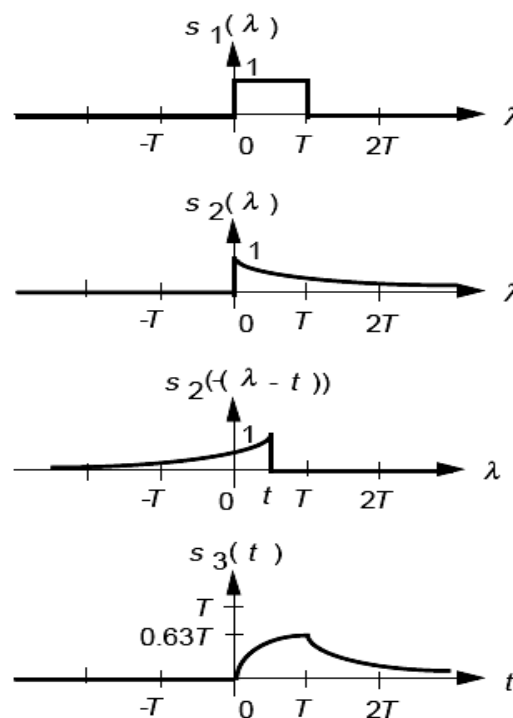
1. Time reversal of  $s_2(t)$  to obtain  $s_2(-\lambda)$ .
2. Time shifting of  $s_2(-\lambda)$  to obtain  $s_2[-(\lambda-t)]$ .
3. Multiplying  $s_1(\lambda)$  and  $s_2[-(\lambda-t)]$  to form the integrand  $s_1(\lambda) s_2[-(\lambda-t)]$ .

Example 2.1 : Convolution of a rectangular waveform

$$s_1(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{elsewhere} \end{cases}$$

with an exponential waveform

$$s_2(t) = e^{-t/T} u(t).$$



Convolution of a rectangular waveform and an exponential waveform.

## 2-5 Review of Fourier Series :

### 2-5-1 The Time-Frequency Concept:

Consider the following set of time functions  $s(t) = \{3A \sin \omega_0 t, A \sin 2\omega_0 t\}$  . We can represent these functions in different ways by plotting the amplitude versus time  $t$ , amplitude versus angular frequency  $\omega$ , or amplitude versus frequency  $f$ .

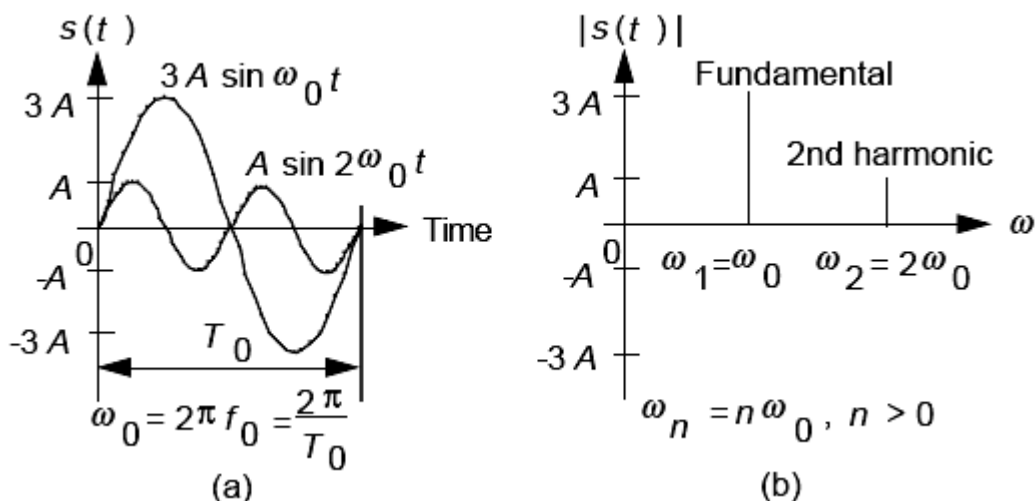


Figure 2.1 (a) Amplitude-time plot, and (b) amplitude-angular frequency plot.

$\omega_0 = 2\pi/T_0$  is called the fundamental angular frequency and  $\omega_2 = 2\omega_0$  is called the second harmonic of the fundamental. In general,  $\omega_n = n\omega_0$  is said to be the  $n$ th harmonic of the fundamental, where  $n > 1$ .

In communication engineering we are interested in steady-state analysis much of the time.

The Fourier series provides a useful model for analysing the frequency content and the steady-state network response for periodic input signals.

### 2-5-2 Trigonometric (Quadrature) Fourier Series :

A periodic time function  $s(t)$  over the interval

$$a - \frac{T_0}{2} < t < a + \frac{T_0}{2}$$

may be represented by an infinite sum of sinusoidal waveforms

$$s(t) = \frac{a_0}{T_0} + \frac{2}{T_0} \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad (2.1)$$

where  $T_0$  is the period of the fundamental frequency  $f_0$  and  $f = 1/T_0$ . This is called the trigonometric (quadrature) Fourier series representation of the time function  $s(t)$ . The coefficients  $a_n$  and  $b_n$  are given by :

$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} s(t) \cos n\omega_0 t dt, n \geq 0 \quad (2.2)$$

and

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} s(t) \sin n\omega_0 t dt, n > 0 \quad (2.3)$$

The choice of  $a_0$  is arbitrary, and it is usually set to 0.

Many forms of the trigonometric Fourier series may be written. For example,

$$s(t) = a'_0 + \sum_{n=1}^{\infty} (a'_n \cos n\omega_0 t + b'_n \sin n\omega_0 t) \quad (2.4)$$

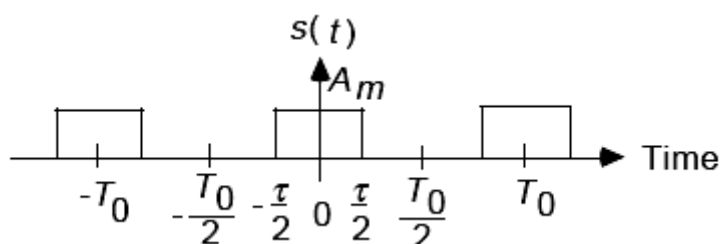
is commonly used. The coefficients  $a'_n$  and  $b'_n$  are given by :

$$a'_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} s(t) \cos n\omega_0 t dt, n \geq 0 \quad (2.5)$$

and

$$b'_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} s(t) \sin n\omega_0 t dt, n > 0 \quad (2.6)$$

**Example 2.2** : Find the trigonometric Fourier series for the periodic time function  $s(t)$  shown in Figure 2.2.



**Figure 2.2** A periodic rectangular waveform.

$$a_n = \int_{-T_0/2}^{T_0/2} s(t) \cos n\omega_0 t \, dt$$

$$a_n = \int_{-\tau/2}^{\tau/2} A_m \cos n\omega_0 t \, dt$$

$$a_n = 2A_m \frac{\sin n\omega_0 \tau / 2}{n\omega_0} = A_m \tau \frac{\sin n\omega_0 \tau / 2}{n\omega_0 \tau / 2}$$

$$b_n = \int_{-T_0/2}^{T_0/2} s(t) \sin n\omega_0 t \, dt = 0$$

Therefore ,

$$s(t) = \frac{A_m \tau}{T_0} + \frac{2}{T_0} \sum_{n=1}^{\infty} \left( A_m \tau \frac{\sin n\omega_0 \tau / 2}{n\omega_0 \tau / 2} \right) \cos n\omega_0 t.$$

### 2-5-3 Exponential (Complex or Phasor) Fourier Series :

The time function  $s(t)$  may be represented over the interval

$$a - \frac{T_0}{2} < t < a + \frac{T_0}{2}$$

by the equivalent exponential (complex or phasor) Fourier series

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad (2.7)$$

where the coefficients  $c_n$  are given by :

$$c_n = \int_{a - T_0/2}^{a + T_0/2} s(t) e^{-jn\omega_0 t} dt \quad (2.8)$$

$c_0$  is equivalent to the dc value of the waveform  $s(t)$ .

$c_n$  is, in general, a complex number. Furthermore, it is a phasor since it is the coefficient of  $e^{jn\omega_0 t}$ .

The complex Fourier series is easier to use for analytical problems.

Many forms of the complex Fourier series may be written. For example,

$$s(t) = \sum_{n=-\infty}^{\infty} c'_n e^{jn\omega_0 t} \quad (2.9)$$

is commonly used. The coefficients  $c'_n$  are given by :

$$c'_n = \frac{1}{T_0} \int_{a - T_0/2}^{a + T_0/2} s(t) e^{-jn\omega_0 t} dt \quad (2.10)$$

**Example 2.3:** Find the complex Fourier series for the periodic time function  $s(t)$  shown in Figure 2.2.

$$c_n = \int_{-T_0/2}^{T_0/2} s(t) e^{-jn\omega_0 t} dt$$

$$c_n = \int_{-\tau/2}^{\tau/2} A_m e^{-jn\omega_0 t} dt$$

$$c_n = A_m \frac{e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}}{jn\omega_0}$$

$$c_n = 2A_m \frac{\sin n\omega_0 \tau/2}{n\omega_0} = A_m \tau \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2}$$

Therefore,

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \left( A_m \tau \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2} \right) e^{jn\omega_0 t}$$

The frequency spectrum is shown in Figure 2.3.

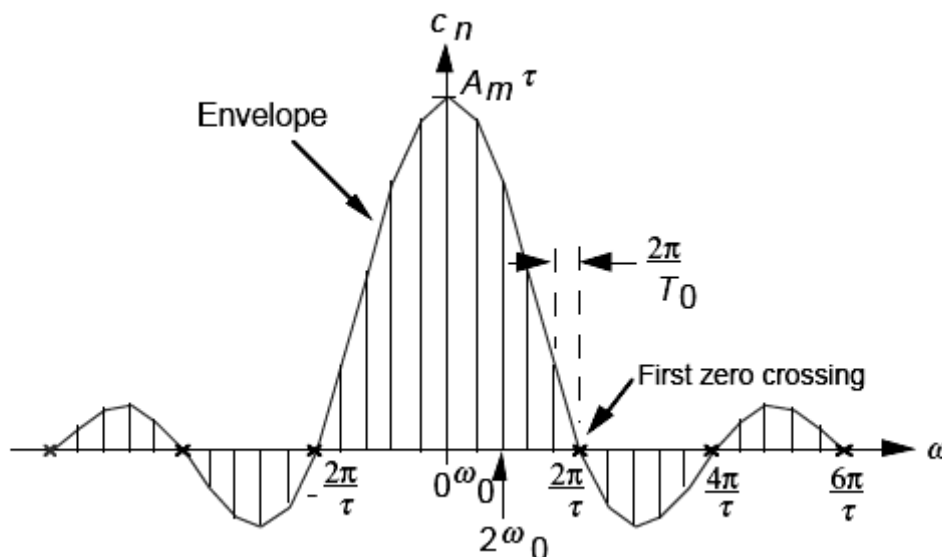


Figure 2.3 Frequency spectrum of a periodic rectangular waveform.

Figure 2.4 shows the effect on the frequency spectrum of smaller  $\tau$

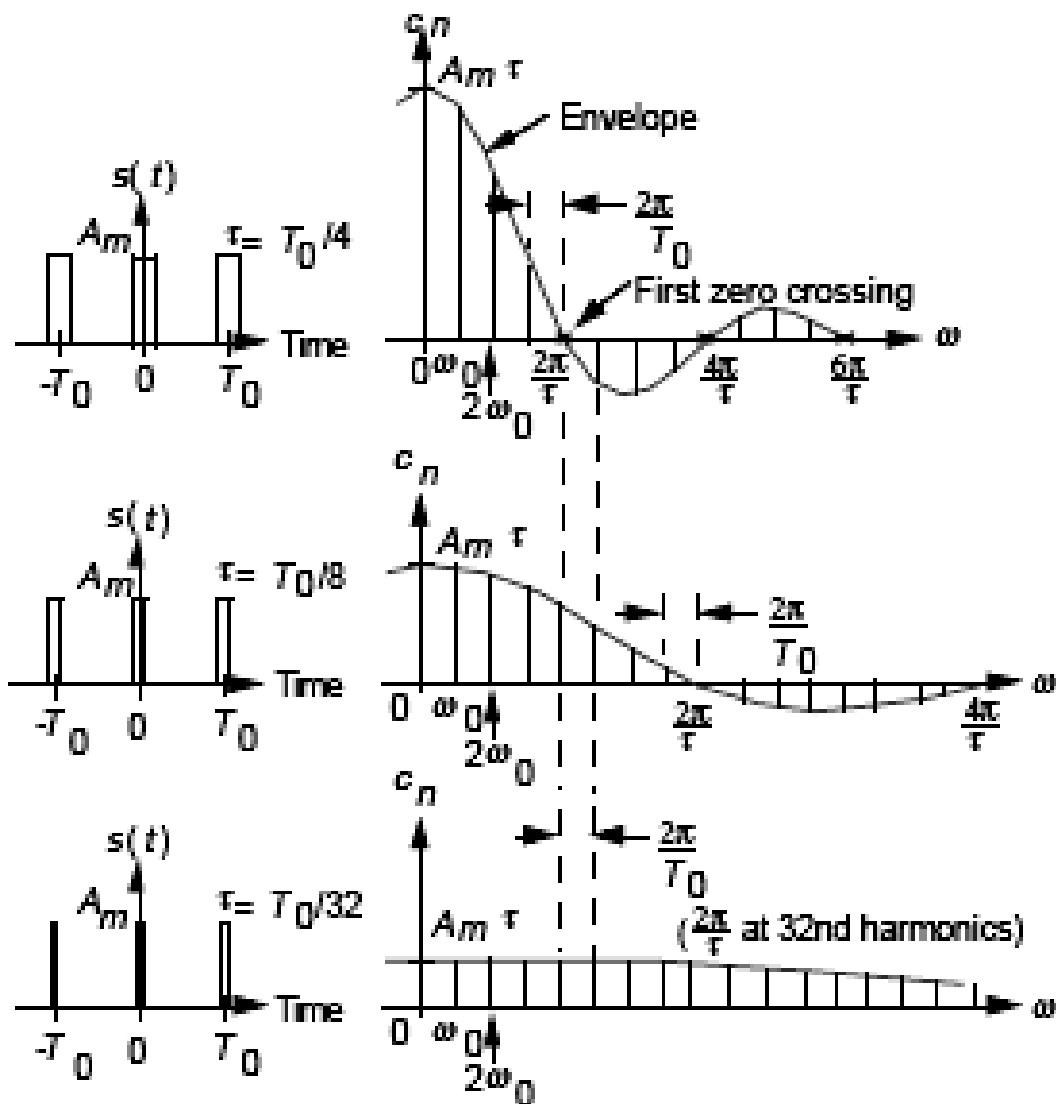
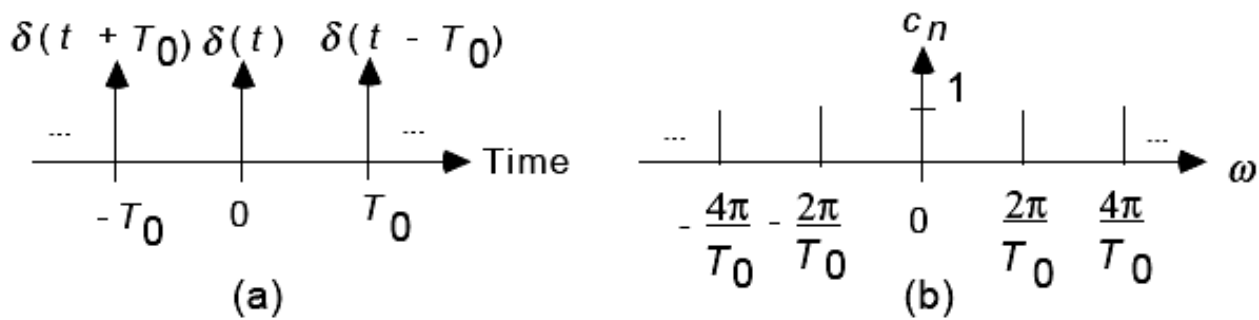


Figure 2.4 Effect on frequency spectrum of smaller  $\tau$ .

If the bandwidth  $B$  is specified as the width of the frequency band of a waveform from zero frequency to the first zero crossing, then  $B = 1/\tau$  Hz. If we let the pulse width  $\tau$  in Figure 2.4 go to zero and the amplitude  $A_m$  go to infinity with  $A_m \tau = 1$ , all spectral lines in the frequency domain have unity length. Figure 2.5 shows the periodic unit impulses and the frequency spectrum of the periodic unit impulses. The bandwidth becomes infinite.





**Figure 2.5** (a) Periodic unit impulses, and (b) frequency spectrum.

**Properties of the Complex Fourier Series :**

1. If  $s(t)$  is real, then  $c_n = c_{-n}^*$
2. If  $s(t)$  is real and even,  $s(t) = s(-t)$ , then  $\text{Im}[c_n] = 0$
3. If  $s(t)$  is real and odd,  $s(t) = -s(-t)$ , then  $\text{Re}[c_n] = 0$
4. The complex Fourier-series coefficients of a real waveform are related to the quadrature Fourier-series coefficients by :

$$c_n = \begin{cases} a_n - jb_n, & n > 0 \\ a_0, & n = 0 \\ a_{-n} + jb_{-n}, & n < 0 \end{cases}$$

$$|c_n| = \sqrt{a_n^2 + b_n^2}$$

represents the amplitude spectrum and

$$\angle c_n = \theta_n = \tan^{-1} \frac{-b_n}{a_n}$$

represents the phase spectrum of the real waveform.

The equivalence between the Fourier series coefficients is demonstrated in Figure 2.6.

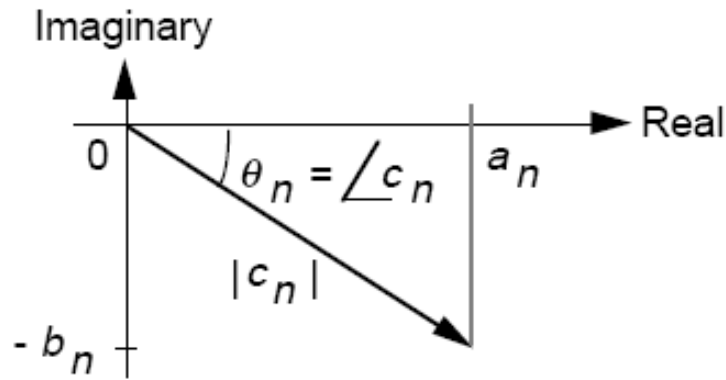


Figure 2.6 Fourier series coefficients,  $n > 1$ .

### Parseval's Theorem for the Fourier Series :

Parseval's Theorem for the Fourier series states that, if  $s(t)$  is a periodic signal with period  $T_0$ , then the average normalized power (across a  $1\Omega$  resistor) of  $s(t)$  is :

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt = \sum_{n=-\infty}^{\infty} \left| \frac{c_n}{T_0} \right|^2$$

If  $s(t)$  is real,  $|s(t)|$  is simply replaced by  $s(t)$ .

### References

- [1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw- Hill, 1990.
- [2] P. H. Young, Electronic Communication Techniques, 4/e, Prentice-Hall, 1998.
- [3] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.
- [4] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

Problems :

1- Find the F.S. expansion of the signal defined by :

$$f(t) = \begin{cases} V & 0 \leq \omega t \leq \pi \\ -V & \pi \leq \omega t \leq 2\pi \end{cases}$$

and plot its amplitude spectrum .

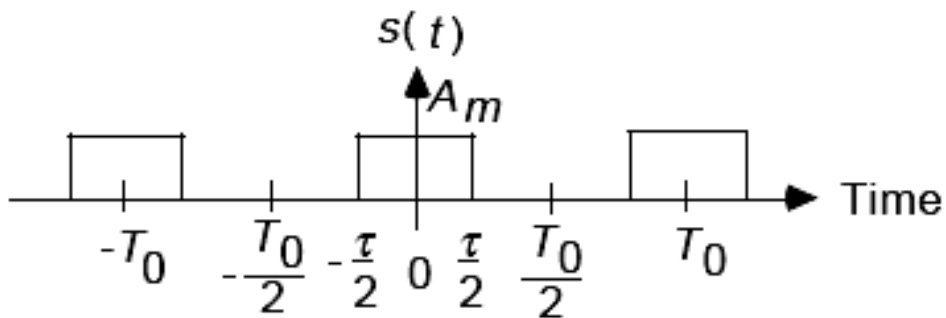
2-What is the F.S. expansion of the periodic signal whose definition in one period is :

$$s(t) = \begin{cases} 0 & -\pi \leq \omega t \leq 0 \\ \sin \omega t & 0 \leq \omega t \leq \pi \end{cases}$$

Plot its amplitude spectrum .

3-What percentage of the total power is contained within the first zero crossing of the spectrum envelope for  $s(t)$  as shown in the following figure.

Assume that  $A_m = 1 \text{ v}$  ,  $T_0 = 0.25 \text{ msec.}$  , and  $\tau = 0.05 \text{ msec}$



## 2-6. Fourier Transform :

In communication systems, we often deal with non-periodic signals. An extension of the time-frequency relationship to a non-periodic signal  $s(t)$  requires the introduction of the Fourier Integral. A nonperiodic signal can be viewed as a limiting case of a periodic signal, where the period  $T_0$  approaches infinity. As  $T_0$  approaches infinity, the periodic signal will eventually become a single non-periodic signal. This is shown in Figure 2-7 .

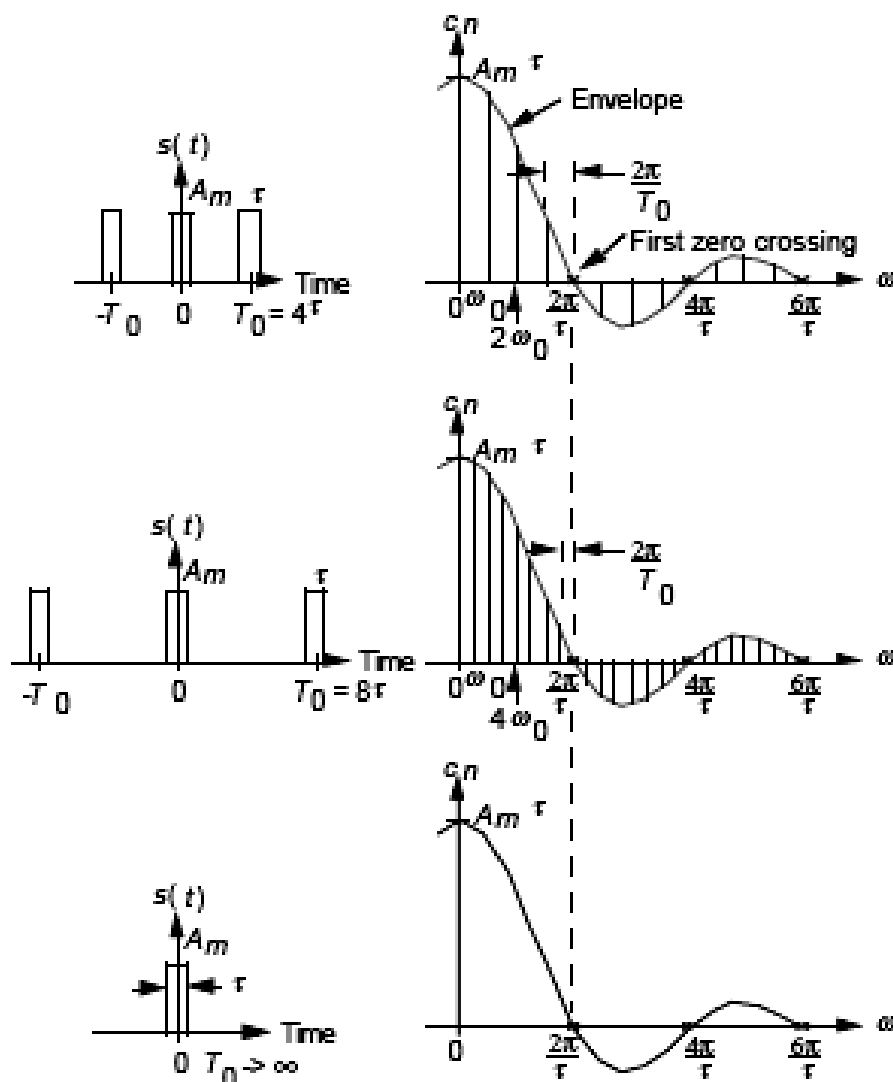


Figure 2.7 Effect on frequency spectrum of increasing period  $T_0$  .

The normalized energy of the non-periodic signal becomes finite and its normalized power tends to zero.

Consider the amplitude spectrum of a periodic waveform as shown in Figure 2-8 .

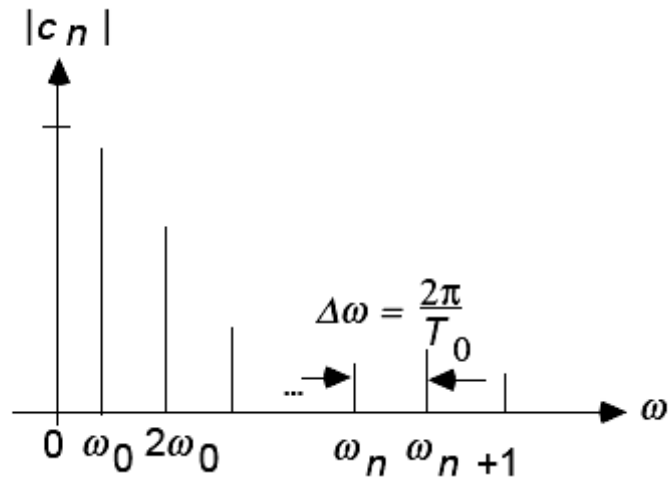


Figure 2.8 Amplitude spectrum of a periodic time function.

Let  $\omega_n = n\omega_0$  and  $\Delta\omega = \omega_{n+1} - \omega_n = 2\pi / T_0$  . The Fourier series of a periodic waveform  $s(t)$  with period  $T_0$  can be written as :

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \Delta\omega$$

and

$$c_n = \int_{-T_0/2}^{T_0/2} s(t) e^{-jn\omega_0 t} dt$$

If  $T_0$  approaches infinity,  $\omega_0$  goes to 0. The harmonics get closer and closer together. In the limit, the Fourier series summation representation of  $s(t)$  becomes an integral,  $c_n$  becomes a continuous function  $S(\omega)$ , and we have a continuous frequency spectrum. In summary, as  $T_0 \longrightarrow \infty$  ,  $\sum$  becomes  $\int$  ,  $\omega_n$  becomes  $\omega$ , and  $\Delta\omega$  becomes  $d\omega$ . We have :

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} c_n e^{j\omega t} d\omega$$

and

$$S(\omega) = F[s(t)] = \lim_{T \rightarrow \infty} \int_0^T s(t) e^{-j\omega t} dt$$

It is also very common to work in terms of frequency  $f$ ,  $f = \omega/2\pi$ , because spectrum analyzers are usually calibrated in hertz. Thus,

$$s(t) = F^{-1}[S(f)] = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft} df$$

and

$$S(f) = F[s(t)] = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt$$

The functions  $s(t)$  and  $S(f)$  are said to constitute a Fourier transform pair, where  $S(f)$  is the Fourier transform of a time function  $s(t)$ , and  $s(t)$  is the Inverse Fourier transform (IFT) of a frequency-domain function  $S(f)$ .

Shorthand notation expressed in terms of  $t$  and  $f$  :  $s(t) \longleftrightarrow S(f)$

Shorthand notation expressed in terms of  $t$  and  $\omega$  :  $s(t) \longleftrightarrow S(\omega)$

In general,  $S(f)$  is a complex function of frequency.

In two-dimensional Cartesian form,  $S(f)$  can be expressed as :

$$S(f) = X(f) + jY(f)$$

In polar form,  $S(f)$  can be expressed as :

$$S(f) = |S(f)| e^{j\theta(f)}$$

Where

$$|S(f)| = \sqrt{X^2(f) + Y^2(f)} \quad \text{and} \quad \theta(f) = \tan^{-1} \frac{Y(f)}{X(f)}$$

$|S(f)|$  represents the amplitude spectrum and  $\theta(f)$  represents the phase spectrum of  $s(t)$ .

Example 2-4 : Find the spectrum of an exponential pulse

$$s(t) = \begin{cases} e^{-t}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$S(f) = \int_0^{\infty} e^{-t} e^{-j2\pi ft} dt$$

$$S(f) = \frac{1}{1+j2\pi f}$$

$$|S(f)| = \sqrt{\frac{1}{1+(2\pi f)^2}}$$

$$\theta(f) = -\tan^{-1}(2\pi f)$$

Transforms of Some Useful Functions :

1. Dirac Delta Time Function :

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt = 1$$

$$\delta(t) \longleftrightarrow 1$$

Also, it can be shown that  $\delta(t-t_0) \longleftrightarrow e^{-j2\pi ft_0}$

2. Dirac Delta Frequency-Domain Function :

$$F^{-1}[\delta(f)] = \int_{-\infty}^{\infty} \delta(f) e^{j2\pi ft} df = 1$$

$$1 \longleftrightarrow \delta(f)$$

Also, it can be shown that  $e^{j2\pi f_0 t} \longleftrightarrow \delta(f - f_0)$

Example 2.5 : Find the spectrum of a sinusoid :

$$v(t) = A \sin 2\pi f_0 t = A(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}) / 2j$$

Since  $e^{j2\pi f_0 t} \longleftrightarrow \delta(f - f_0)$ , we have

$$V(f) = \frac{A}{2j} \delta(f - f_0) - \frac{A}{2j} \delta(f + f_0)$$

$$V(f) = -\frac{A}{2}j[\delta(f - f_0) - \delta(f + f_0)]$$

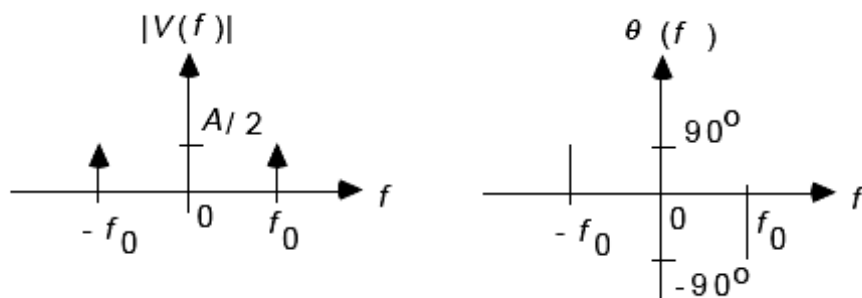
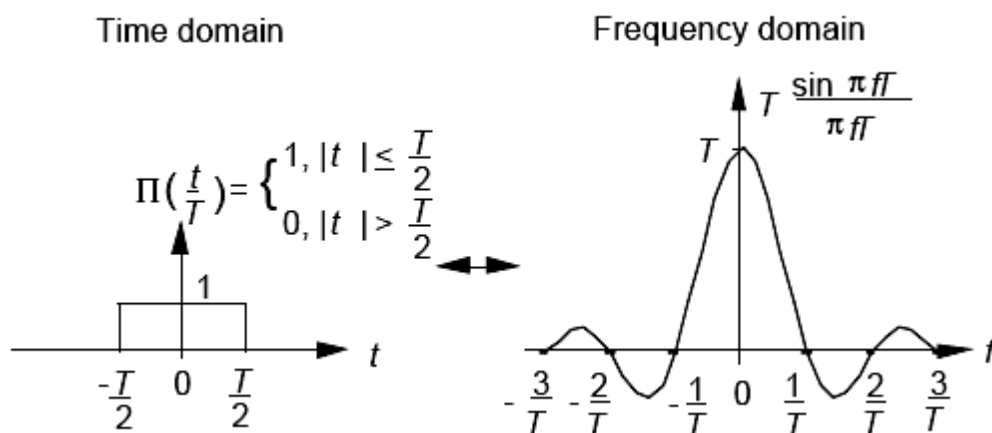


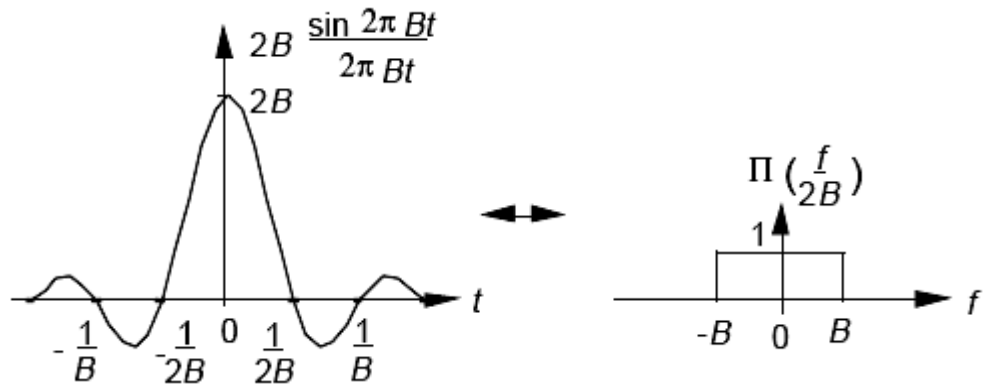
Figure 2.9 Spectrum of the periodic function  $A \sin 2\pi f_0 t$

3. Rectangular,  $\sin x/x$ , and Triangular Pulses :

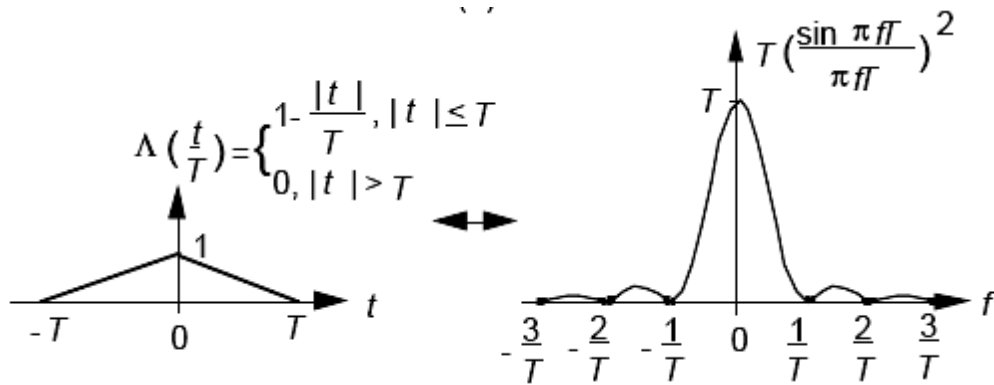


(a)





(b)



(c)

Figure 2.10 Spectra of (a) rectangular, (b)  $\sin x/x$ , and (c) triangular pulses.

Observations:

1. Figure 2.10-a - Spectrum spreads out as the pulse width  $T$  decreases. Bandwidth  $B = 1/T$  Hz and  $S(f)$  decreases as  $1/f$ .

2. Figure 2.10c - spectrum spreads out as the pulse width  $T$  decreases. Bandwidth  $B = 1/T$  Hz and  $S(f)$  decreases as  $1/f^2$ .

The smoother the time-domain function, the more rapidly the spectrum decreases with increasing frequency, packing more frequency contents into a specified bandwidth.

An inverse time-bandwidth relation always exists.

Bandwidth plays a significant role in determining transmission rate.

## Properties of Fourier Transforms :

### 1. Symmetry (Duality) Property :

$$s(t) \longleftrightarrow S(-f)$$

### 2. Scaling Property :

$$s(at) \longleftrightarrow \frac{1}{|a|} S\left(\frac{f}{a}\right)$$

### 3. Time Shifting (Time Delay) Property :

$$s(t - T_d) \longleftrightarrow S(f) e^{-j2\pi f T_d}$$

### 4. Frequency Shifting Property :

$$s(t) e^{j2\pi f_c t} \longleftrightarrow S(f - f_c)$$

### 5. Differentiation Property :

$$d^n s(t) / dt^n \longleftrightarrow (j2\pi f)^n S(f)$$

Differentiation increases the high-frequency content of a signal. The derivative of an even function must be odd. Hence, the Fourier transform of the derivative of the function must be odd and imaginary.

### 6. Convolution Property :

$$s_1(t) * s_2(t) \longleftrightarrow S_1(f) S_2(f)$$

### 7. If $s(t)$ is real, then :

$$S(-f) = S^*(f)$$

### 8. If $s(t)$ is real, then :

$$|S(-f)| = |S(f)|$$

and

$$\theta(-f) = -\theta(f)$$

The following table shows Fourier transform properties for various forms of  $s(t)$ .

If $s(t)$ is a:	Then $S(f)$ is a:
Real and even function of $t$	Real and even function of $f$
Real and odd	Imaginary and odd
Imaginary and even	Imaginary and even
Imaginary and odd	Real and odd
Complex and even	Complex and even
Complex and odd	Complex and odd

Example 2.6: Use the scaling and real-signal frequency-translation properties to find the Fourier transform of a damped sinusoid

$$s(t) = \begin{cases} e^{-t/T} \sin \omega_0 t, & t > 0, T > 0 \\ 0, & t < 0 \end{cases}$$

From Example 2.4 we have :

$$\begin{cases} e^{-t}, & t > 0 \\ 0, & t < 0 \end{cases} \longleftrightarrow \frac{T}{1+j 2\pi f T}$$

Using the scaling property with  $a = 1/T$ , we get :

$$\begin{cases} e^{-t/T}, & t > 0 \\ 0, & t < 0 \end{cases} \longleftrightarrow \frac{T}{1+j 2\pi f T}$$

Using the real-signal frequency-translation property with  $\theta = -\pi/2$ , we get:

$$S(f) = \frac{1}{2} \left[ e^{-j\pi/2} \frac{T}{1+j2\pi(f-f_0)T} + e^{j\pi/2} \frac{T}{1+j2\pi(f+f_0)T} \right]$$

The  $\sin \omega_0 t$  factor causes the spectrum to move from  $f = 0$  to  $f = \pm f_0$ .

## Parseval's Theorem for the Fourier Transform and Energy Spectral Density :

Parseval's Theorem for the Fourier transform states that if  $s_1(t)$  and  $s_2(t)$  are two complex energy signals, then :

$$\int_{-\infty}^{\infty} s_1(t) s_2^*(t) dt = \int_{-\infty}^{\infty} S_1(f) S_2^*(f) df$$

If  $s_1(t) = s_2(t)$ , then **Rayleigh's energy theorem** states that the normalised energy is :

$$E = \int_{-\infty}^{\infty} |s_1(t)|^2 dt = \int_{-\infty}^{\infty} |S_1(f)|^2 df$$

The energy spectral density (ESD) is defined for energy waveforms by:

$$E_{11}(f) = |S_1(f)|^2$$

## Power Spectral Density (PSD) and Wiener-Khintchine Theorem :

PSD is a function showing the distribution of power in the signal as a function of frequency .

The Wiener-Khintchine theorem states that the power spectral density and the autocorrelation function are Fourier transform pairs.

$$R_{22}(\tau) \longleftrightarrow P_{22}(f)$$

Where  $P_{22}(f)$  and  $R_{22}(\tau)$  are the power spectral density and the autocorrelation of a power waveform  $s_2(t)$  respectively .

Furthermore, the average normalized power is :

$$P = \langle s_2^2(t) \rangle = S_{2rms}^2 = \int_{-\infty}^{\infty} P_{22}(f) df = R_{22}(0)$$

The average normalized power of a power waveform is now related to the power spectral density.

The power spectral density (PSD) for a power waveform  $s_2(t)$  is :

$$P_{22}(f) = \lim_{T \rightarrow \infty} \left( \frac{|S_1(f)|^2}{T} \right)$$

where  $S_1(f)$  is the Fourier transform of the truncated waveform  $s_1(t)$  defined as :

$$s_1(t) = \begin{cases} s_2(t), & -T/2 < t < T/2 \\ 0, & \text{elsewhere} \end{cases} = s_2(t) \Pi\left(\frac{t}{T}\right)$$

and  $s_1(t)$  is an energy waveform as long as  $T$  is finite.

The power spectral density is always a real nonnegative function of frequency. It is not sensitive to the phase spectrum of the truncated waveform  $s_1(t)$ . Thus,  $A \sin 2\pi f_0 t$  and  $A \cos 2\pi f_0 t$  have the same PSD because the phase has no effect on the power spectral density.

### Fourier Transform of Periodic Signals :

So far we have used the Fourier series and the Fourier transform to represent periodic and nonperiodic signals, respectively. For periodic signals, we can use an impulse function in the frequency domain to represent discrete components of periodic signals using Fourier transforms. With this approach, both periodic and nonperiodic signals can be incorporated in a common Fourier-transform framework.

### Recall:

$$\begin{aligned} A\delta(t) &\longleftrightarrow A \\ A\delta(t - t_0) &\longleftrightarrow Ae^{-j2\pi f t_0} \\ A &\longleftrightarrow A\delta(f) \\ Ae^{j2\pi f_0 t} &\longleftrightarrow A\delta(f - f_0) \end{aligned}$$

The complex Fourier series of a periodic signal is given by :

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

and the Fourier transform of  $s(t)$  is :

$$S(f) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_0)$$

Example 2.7 : The complex Fourier series of a periodic rectangular waveform  $s(t)$  is :

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \leftrightarrow \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_0)$$

Where :

$$c_n = A_m \tau \left( \frac{\sin 2\pi n f_0 \tau / 2}{2\pi n f_0 \tau / 2} \right).$$

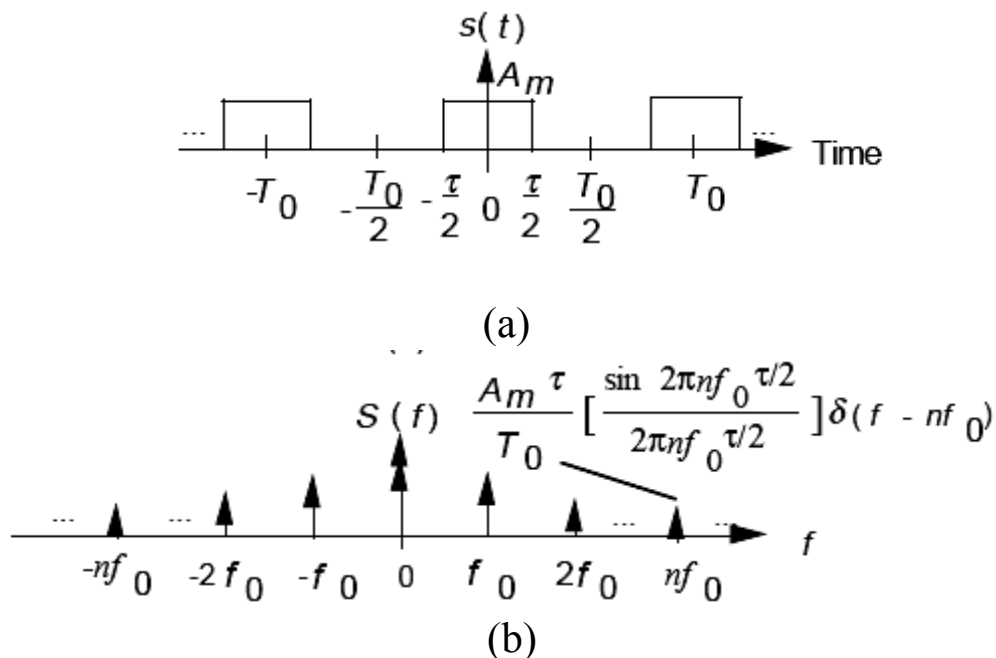
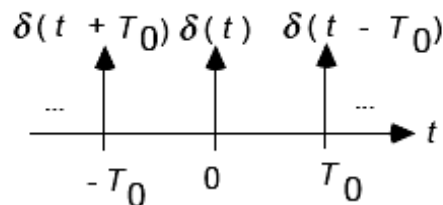


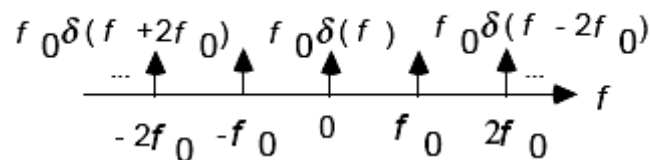
Figure 2.11 (a) A periodic rectangular waveform  $s(t)$ , and (b) the Fourier transform spectrum of  $s(t)$  .

Example 2.8 : A periodic impulse  $s(t)$  is :

$$s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0) \longleftrightarrow f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0).$$



(a)



(b)

Figure 2.12 (a) Periodic impulse  $s(t)$ , and (b) Fourier transform spectrum of  $s(t)$ .

References :

- [1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.
- [2] J. D. Gibson, Modern Digital and Analog Communications, 2/e, Macmillan Publishing Company, 1993.
- [3] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.
- [4] B. P. Lathi, Modern Digital and Analog Communication Systems, 3/e, Oxford University Press, 1998.
- [5] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

## 2.7 Systems :

A system is a mathematical model that relates the output signal to the input signal of a physical process.

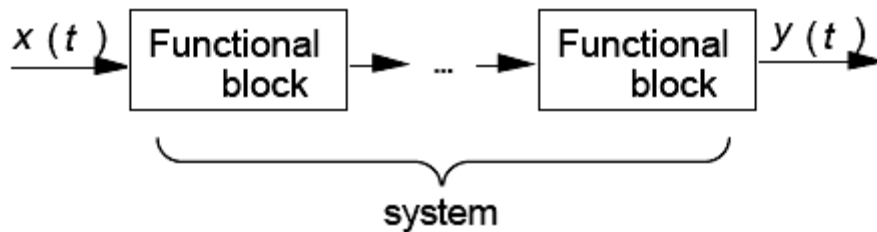


Figure 2.13 Representation of a system.

### Classification of Systems :

#### 1. Linear and non-linear systems

Let  $x_i(t)$  and  $y_i(t)$ ,  $i > 1$ , be input and output signals of a system, respectively. A system is called a linear system if the input  $x_1(t) + x_2(t) + \dots + x_i(t) + \dots$  produces a response  $y_1(t) + y_2(t) + \dots + y_i(t) + \dots$ , and  $ax_i(t)$  produces  $ay_i(t)$  for all input signals  $x_i(t)$  and scalar  $a$ . This is known as the superposition theorem and a linear system obeys this principle.

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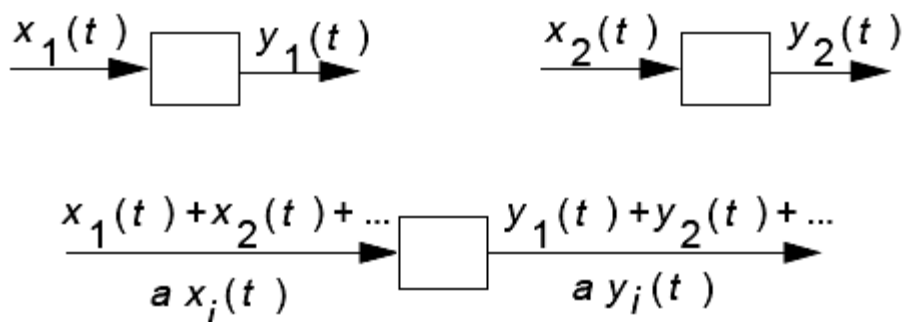


Figure 2.14 Linear system.

In practice, it may be found that a system is only linear over a limited range of input signals.

A non-linear system does not obey the superposition theorem.



## 2.Causal and non-causal systems :

Let  $x(t)$  and  $y(t)$  be the input and output signals of a system. A causal (physically realizable) system produces an output response at time  $t_1$  for an input at time  $t_0$ , where

$$t_0 \geq 0 \text{ and } t_0 \leq t_1.$$

In other words, a causal system is one whose response does not begin before the input signal is applied.

A non-causal system response will begin before the input signal is applied. It can be made realizable by introducing a positive time delay into the system .

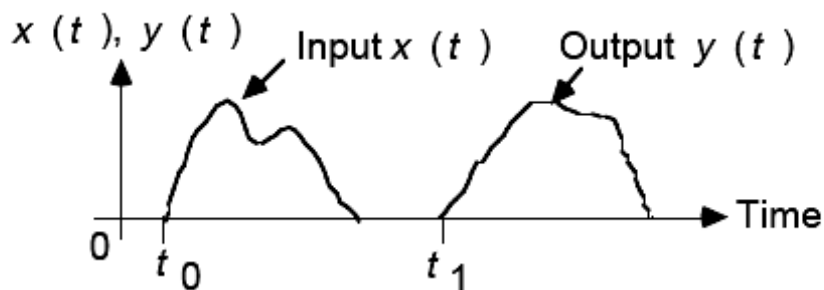


Figure 2.15 Signals associated with causal system.

## 3.Time-invariant and time-varying systems :

If the input  $x(t - t_0)$  produces a response  $y(t - t_0)$  where  $t_0$  is any real constant, the system is called a time-invariant system.

If the above condition is not satisfied, the system is called a time-varying system.

A system is called a linear time-invariant (LTI) system if the system is linear and time-invariant.

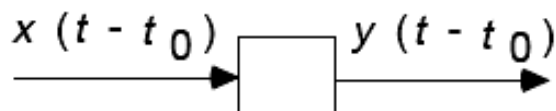


Figure 2.16 Time-invariant system.

Classification of signals and systems will help us in finding a suitable mathematical model for a given physical process that is to be analyzed.

## Impulse Response :

The impulse response  $h(t)$  of an LTI system is defined as the response of the system when the input signal  $x(t)$  is a delta function  $\delta(t)$ . The output  $y(t)$  of an LTI system can be expressed as the convolution of the input signal  $x(t)$  and the impulse response  $h(t)$  of the system, i.e.,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda$$
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda) x(t-\lambda) d\lambda$$

For a causal LTI system :

$$y(t) = \int_0^{\infty} x(\lambda) h(t-\lambda) d\lambda = \int_0^{\infty} h(\lambda) x(t-\lambda) d\lambda$$

## Transfer Function :

In the frequency domain, the Fourier transform of  $y(t) = h(t) * x(t)$  is :

$$Y(f) = H(f) X(f)$$

where  $H(f)$  is the Fourier transform of  $h(t)$ .  $H(f)$  is called the transfer function or frequency response of the LTI system.

Example 2.9 : An RC circuit is shown in Figure 2.17 , using Kirchhoff's voltage law, we get :

$$x(t) = i(t) R + y(t)$$

Since  $i(t) = C \frac{dy}{dt}$ , we can write

$$RC \frac{dy}{dt} + y(t) = x(t)$$

and

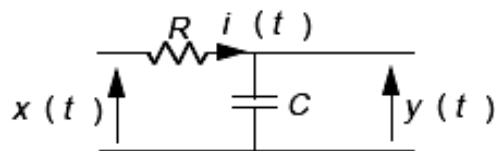
$$RC(j2\pi f)Y(f) + Y(f) = X(f)$$

The transfer function of the  $RC$  circuit is:

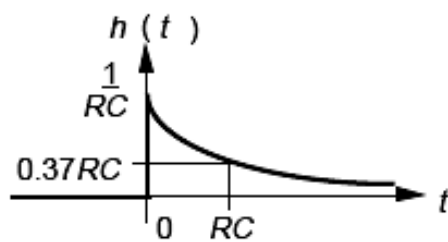
$$H(f) = \frac{1}{1 + j 2\pi RC f}$$

and the impulse response is :

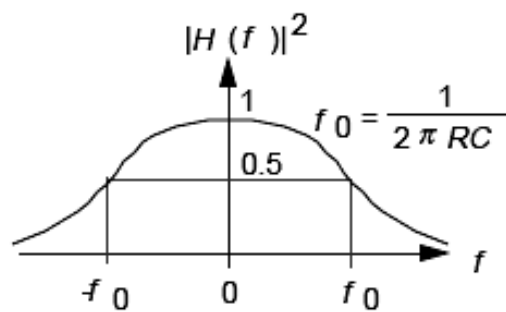
$$h(t) = \begin{cases} \frac{1}{RC} e^{-\frac{t}{RC}}, & t \geq 0, \\ 0, & t < 0 \end{cases}$$



(a)



(b)



(c)

Figure 2.17 Characteristics of an  $RC$  circuit.

Consider the input signal  $x(t) = \delta(t)$ . The Fourier transform of  $x(t)$  is

$$X(f) = F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j 2\pi f t} dt = 1$$

and  $Y(f) = H(f)$ .

$X(f) = 1$  implies equal amplitude at all frequencies. It is equivalent to exciting the system with all frequencies simultaneously.

Rather than applying a sinusoidal signal and varying its frequency continuously to obtain  $H(f)$ , a technique to measure  $H(f)$  is as follows:

1. Excite the system with  $x(t) = \delta(t)$ .
2. Measure  $y(t) = h(t)$ .
3. Find :

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j 2\pi f t} dt.$$

In general,  $H(f)$  is a complex function of frequency. In polar form,  $H(f)$  can be expressed as  $H(f) = |H(f)| e^{j\theta(f)}$

From our earlier study of the properties of Fourier transform, we have seen that if  $h(t)$  is real,

$$H(-f) = H^*(f)$$

$$|H(-f)| = |H(f)|$$

$$\theta(-f) = -\theta(f)$$

where  $H^*(f)$  is the complex conjugate of  $H(f)$ . If  $x(t)$  and  $y(t)$  are real and  $H(f)$  is the transfer function of a LTI system, we can obtain the following results.

Let  $x(t)$  and  $y(t)$  be the input and output energy signals of a LTI system. The energy spectral densities of  $x(t)$  and  $y(t)$  are  $E_{xx}(f) = |X(f)|^2$  and

$E_{yy}(f) = |Y(f)|^2$ , respectively.

Since  $Y(f) = H(f) X(f)$ , we have

$$E_{yy}(f) = |H(f)|^2 E_{xx}(f)$$

$$E_{yy}(f) = H^*(f)H(f) E_{xx}(f)$$

Let  $x(t)$  and  $y(t)$  be the input and output power signals of a LTI system. Also, let  $x_T(t)$  and  $y_T(t)$  be the truncated signals of  $x(t)$  and  $y(t)$ , respectively, where

$$x_T(t) = \begin{cases} x(t), & -T/2 < t < T/2 \\ 0, & \text{elsewhere} \end{cases} = x(t) \Pi\left(\frac{t}{T}\right)$$

and

$$y_T(t) = \begin{cases} y(t), & -T/2 < t < T/2 \\ 0, & \text{elsewhere} \end{cases} = y(t)\Pi\left(\frac{t}{T}\right)$$

$x_T(t)$  and  $y_T(t)$  are energy signals as long as  $T$  is finite. The power spectral densities of  $x(t)$  and  $y(t)$  are :

$$P_{xx}(f) = \lim_{T \rightarrow \infty} \left( \frac{|X_T(f)|^2}{T} \right)$$

and

$$P_{yy}(f) = \lim_{T \rightarrow \infty} \left( \frac{|Y_T(f)|^2}{T} \right)$$

, respectively.

Since  $Y(f) = H(f) X(f)$ , we have

$$P_{yy}(f) = |H(f)|^2 P_{xx}(f)$$

$$P_{yy}(f) = H^*(f)H(f) P_{xx}(f)$$

We can also obtain the relationship between the input and output auto-correlation functions of a LTI system. Since the energy/power spectral density and the auto-correlation function are Fourier transform pairs, the inverse Fourier transform of  $E_{yy}(f)$  or  $P_{yy}(f)$  is :

$$R_{yy}(\tau) = h(-\tau) * h(\tau) * R_{xx}(\tau)$$

where  $R_{xx}(\tau)$  and  $R_{yy}(\tau)$  are the auto-correlation functions of  $x(t)$  and  $y(t)$ , respectively.

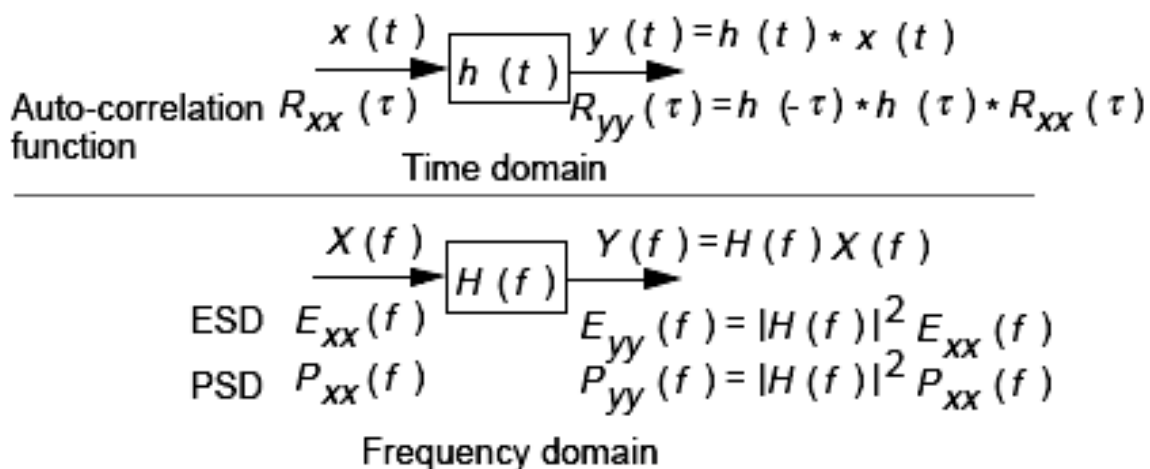


Figure 2.18 Input and output relationships of linear system.

## Distortionless Transmission :

In communication systems, a distortionless transmission is often desired. This implies that the output signal  $y(t)$  is given by :

$$y(t) = K x(t - t_d)$$

where  $K$  is a constant and  $t_d$  is a time delay. The Fourier transform of  $y(t)$  is :

$$\begin{aligned} Y(f) &= K e^{-j2\pi f t_d} X(f) \\ &= H(f) X(f) \end{aligned}$$

where

$$H(f) = K e^{-j2\pi f t_d}$$

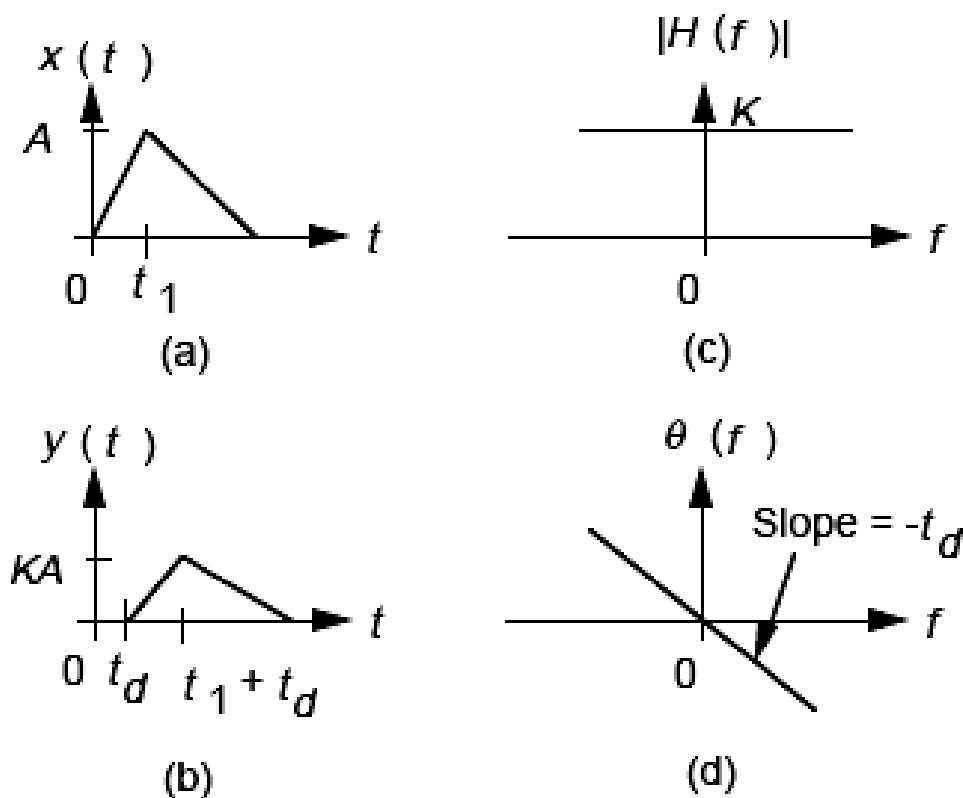


Figure 2.18 Waveforms and spectra associated with distortionless transmission.

## Classification of Filters

### 1. Ideal Low-Pass Filter.

The transfer function of an ideal low-pass filter is defined by:

$$H_{LPPF}(f) = \begin{cases} e^{-j 2\pi f t_d} & \text{for } |f| \leq f_c \\ 0 & \text{elsewhere} \end{cases}$$

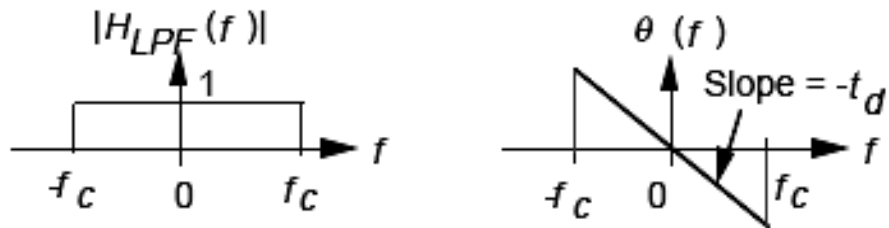


Figure 2.19 Frequency response of an ideal LPF.

The bandwidth of an ideal low-pass filter is equal to  $f_c$ .

### 2. Ideal High-Pass Filter.

The transfer function of an ideal high-pass filter is defined by :

$$H_{HPF}(f) = \begin{cases} e^{-j 2\pi f t_d} & \text{for } |f| \geq f_c \\ 0 & \text{elsewhere} \end{cases}$$

or

$$H_{HPF}(f) = e^{-j 2\pi f t_d} \cdot H_{LPPF}(f)$$

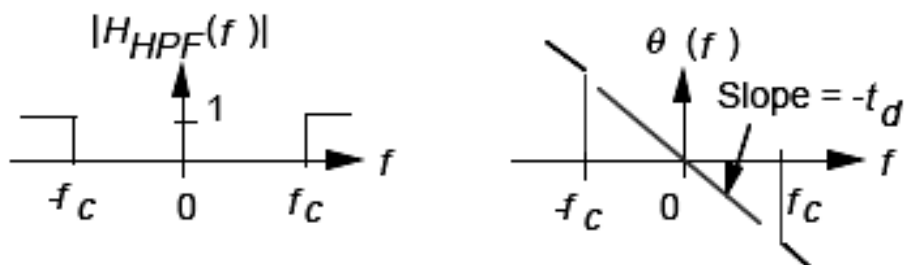


Figure 2.20 Frequency response of an ideal HPF.

The bandwidth of an ideal high-pass filter is not defined, or infinite.

### 3. Ideal Bandpass Filter.

The transfer function of an ideal bandpass filter is defined by:

$$H_{BPF}(f) = \begin{cases} e^{-j 2\pi f t_d} & \text{for } f_{c1} \leq |f| \leq f_{c2} \\ 0 & \text{elsewhere} \end{cases}$$

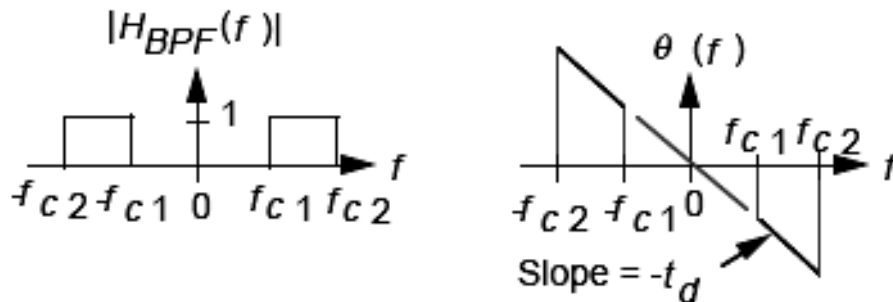


Figure 2.21 Frequency response of an ideal BPF.

The bandwidth of an ideal bandpass filter is equal to  $f_{c2} - f_{c1}$ .

### 4. Ideal Bandstop Filter.

The transfer function of an ideal bandstop filter is defined by:

$$H_{BSF}(f) = \begin{cases} e^{-j 2\pi f t_d} & \text{for } 0 \leq |f| \leq f_{c1} \text{ and } |f| \geq f_{c2} \\ 0 & \text{elsewhere} \end{cases}$$

or

$$H_{BSF}(f) = e^{-j 2\pi f t_d} - H_{BPF}(f)$$

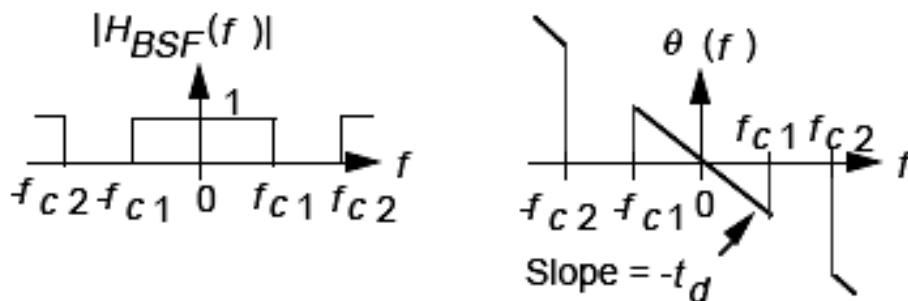


Figure 2.22 Frequency response of an ideal BSF.

The bandwidth of an ideal bandstop filter is not defined, or infinite.



## Concept of Non-Ideal or Practical Filter (System) Bandwidth

A common definition of a non-ideal filter's bandwidth is the 3dB bandwidth. For low-pass filters, the bandwidth is defined as the positive frequency at which the amplitude spectrum  $|H(f)|$  drops to a value equal to  $|H(f)|/\sqrt{2}$ . For bandpass filters, the bandwidth is defined as the difference between the frequencies at which the amplitude spectrum  $|H(f)|$  drops to a value equal to  $|H(f_0)|/\sqrt{2}$ , where  $H(f_0)$  is the peak value of  $|H(f)|$ .

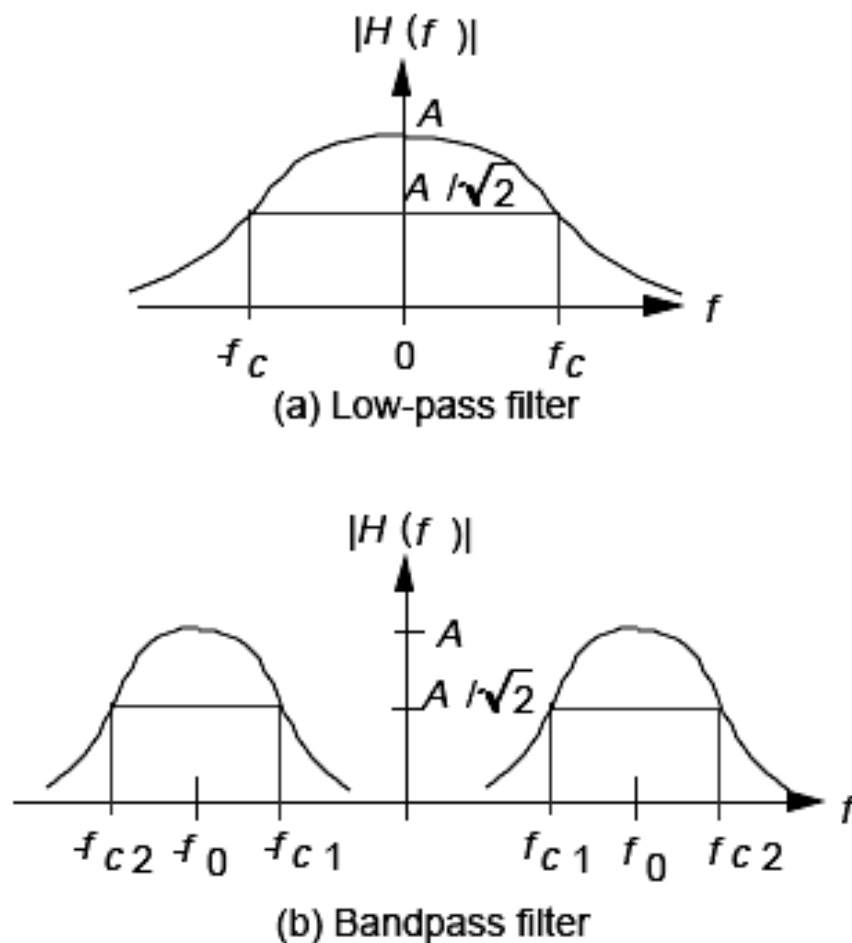


Figure 2.23 Non-ideal filter bandwidth.

## **MODULATION**

Modulation is a process by which some parameter (amplitude, frequency or phase) of a carrier signal is varied in accordance with a message signal. The message signal is called a modulating signal.

A carrier signal, in analog modulation, usually a simple sine wave, contains no information in itself. This gives us three possibilities:

- Amplitude modulation (AM), where the amplitude or strength of the carrier is varied.
- Frequency modulation (FM), where the frequency of the carrier is varied.
- Phase modulation (PM), where the phase of the carrier is varied. It actually turns out that FM and PM are very close relatives.

### **Reasons of modulation:**

- ◆ Match the signal to the channel characteristics and increase the efficiency of information transmission.
- ◆ Shift the frequency band occupied by the message signal.
- ◆ Reduce the relative bandwidth.
- ◆ Separate the signals in the frequency domain.
- ◆ Less sensitivity to channel distortion.

### **Demodulation:**

It is the process of separating the original message (base-band) signal from the noisy and distorted received modulated carrier signal .

### 3. Amplitude Modulation

It is defined as the process of varying the amplitude of a sinusoidal carrier wave in synchronism with , and in direct proportion , to the amplitude of a modulating signal .

The unmodulated carrier signal is given as :

$$s(t) = A \cos 2\pi f_c t$$

A sinusoidal modulated ( band-pass) carrier signal is represented by:

$$s_c(t) = A(t) \cos \theta(t)$$

where  $A(t)$  is the envelope and  $\theta(t) = \omega_c t + \phi(t) = 2\pi f_c t + \phi(t)$ .  $\phi(t)$  is called the instantaneous phase deviation of  $s_c(t)$  and  $f_c$  is the carrier frequency. For amplitude modulation, we can write

$$s_c(t) = A(t) \cos 2\pi f_c t$$

where  $A(t)$  is linearly related to the modulating signal  $m(t)$ .  $A(t)$  is called the instantaneous amplitude of  $s_c(t)$  and amplitude modulation is also referred to as linear modulation. Depending on the relationship between  $m(t)$  and  $A(t)$ , we have the following types of amplitude modulation schemes:

- Normal amplitude modulation (AM) .
- Double-sideband (DSB) modulation .
- Single-sideband (SSB) modulation .
- Vestigial-sideband (VSB) modulation .

## Normal Amplitude Modulation (AM) :

A normal amplitude-modulated signal is given by:

$$s_c(t) = [A + m(t)] \cos 2\pi f_c t$$
$$s_c(t) = \underbrace{A \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{m(t) \cos 2\pi f_c t}_{\text{sidebands}}$$

where  $A$  is a constant and  $m(t)$  is the modulating signal.

The modulation index  $m$  is defined as the ratio of the amplitude of the modulating signal to that of the unmodulated carrier. It is a value between 0 and 1 which describes the “degree of modulation” of the carrier. If  $m = 0$  there is no modulation, while  $m = 1$  is the maximum modulation that can occur without distortion.

Figure 3.1 shows normal AM signals for various values of modulation index. Clearly, the envelope of the modulated signals has the same shape as  $m(t)$  when  $m < 1$ . When  $m > 1$ , the carrier signal is said to be over-modulated and the envelope is distorted.

If  $m(t) = E_m \cos \omega_m t$ , and  $\omega_m = 2\pi f_m$ .

$$A(t) = A + E_m \cos \omega_m t$$

$$s_c(t) = A (1 + m \cos \omega_m t) \cos \omega_c t$$

$$= A \cos \omega_c t + m A \cos \omega_m t \cos \omega_c t$$

$$= A \cos \omega_c t + 0.5 mA \cos(\omega_c - \omega_m)t + 0.5 mA \cos(\omega_c + \omega_m)t$$

and  $m = E_m / A$ .

The total average power of normal AM signal is:

$$P_t = P_c + 0.25 m^2 P_c + 0.25 m^2 P_c$$
$$= P_c (1 + 0.5 m^2)$$

where  $P_c = (A_{rms})^2 / R$  is the power of the unmodulated carrier wave &

$P_s = 0.5 m^2 P_c$  is the power of the upper and lower sidebands.

For 100% modulation,  $m = 1$ ,  $P_t = 1.5 P_c$

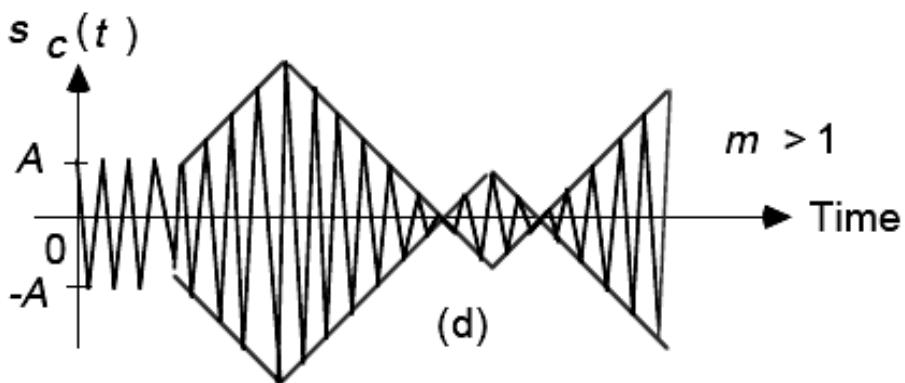
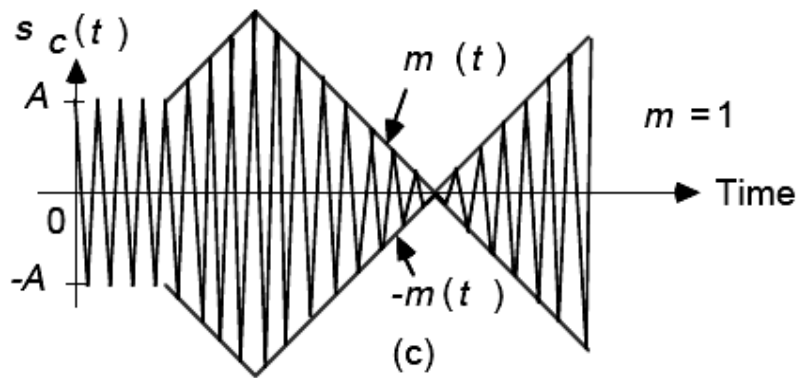
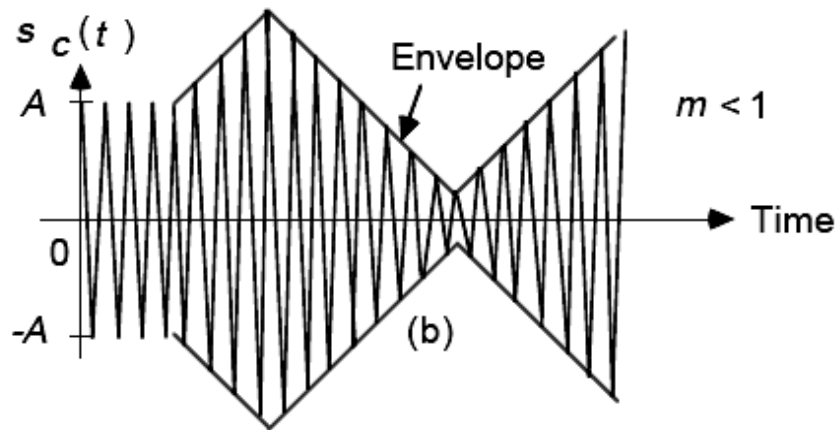
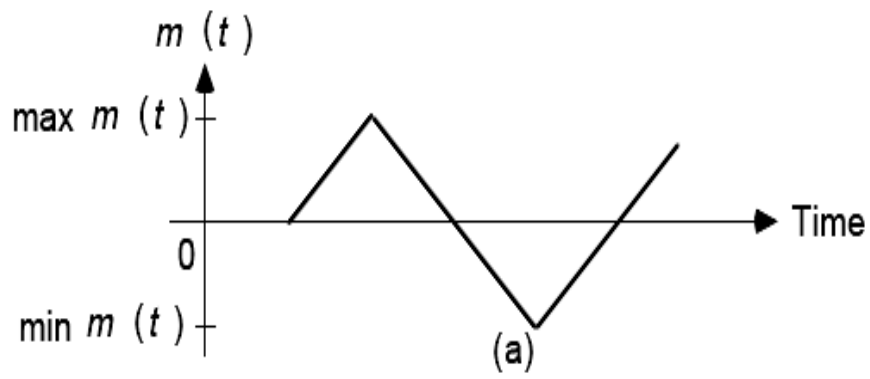


Figure 3.1 Normal AM signals for various values of modulation index

## Spectrum of Normal AM Signals :

For normal amplitude modulation,

$$\begin{aligned} s_c(t) &= [A + m(t)] \cos 2\pi f_c t \\ &= A \cos 2\pi f_c t + m(t) \cos 2\pi f_c t \end{aligned}$$

The Fourier transform of  $s_c(t)$  is :

$$S_c(f) = 0.5 A [\delta(f-f_c) + \delta(f+f_c)] + 0.5 [M(f-f_c) + M(f+f_c)]$$

Normal amplitude modulation simply shifts the spectrum of  $m(t)$  to the carrier frequency  $f_c$ . The bandwidth of the modulated signal is  $2f_m$  Hz, where  $f_m$  is the bandwidth of the modulating signal  $m(t)$ .

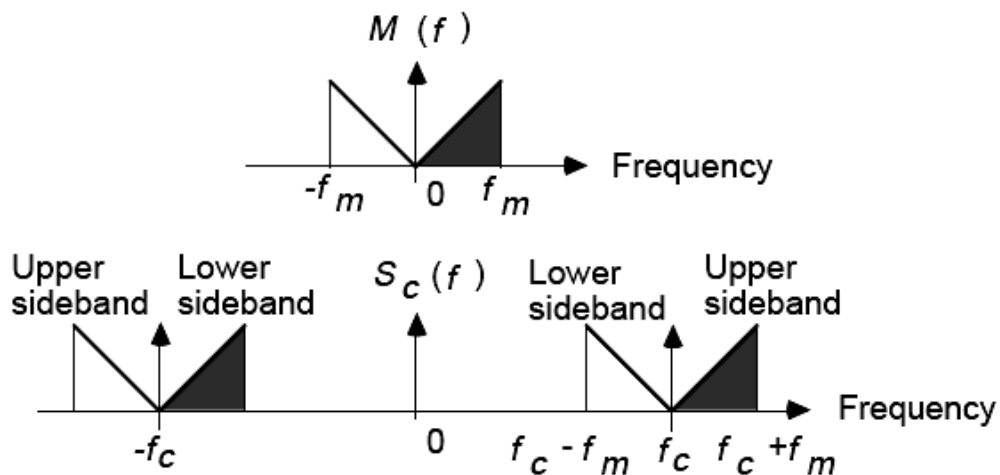


Figure 3.2 Spectrum of normal AM signal.

The efficiency  $\eta$  of a normal AM signal is defined as :

$$\eta = \frac{P_s}{P_t} \times 100\%$$

where  $P_s$  is the power carried by the sidebands and  $P_t$  is the total power of the normal AM signal.

## Generation of Normal AM Signals :

A process of generating a normal AM signal is shown in Figure 3.3. This type of modulation can be achieved by using a non-linear device, such as a diode. This is shown in Figure 3.4 .

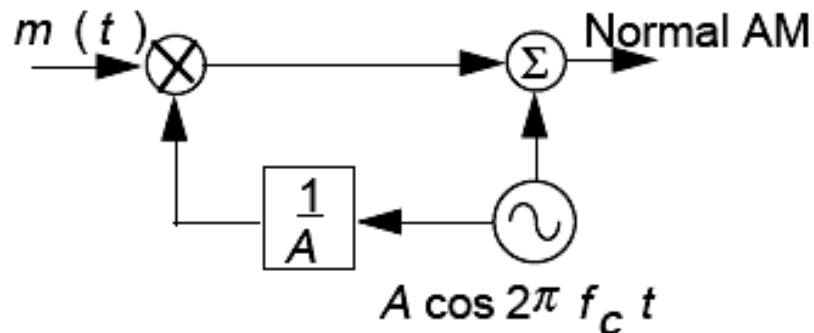


Figure 3.3 Generation of normal AM signal.

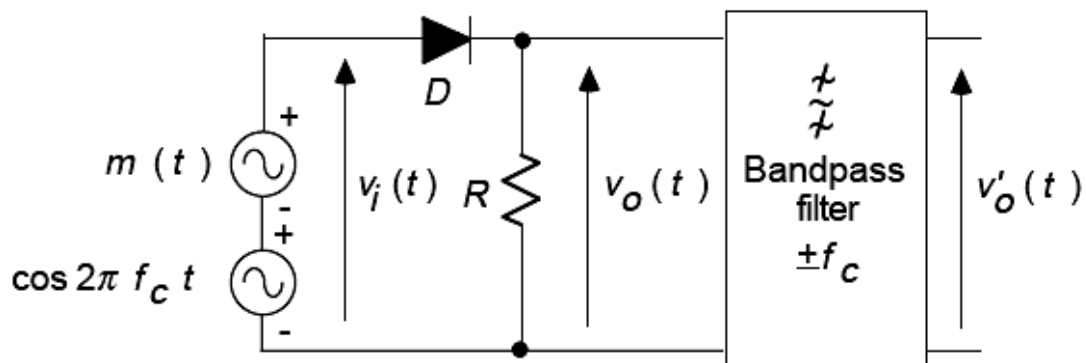


Figure 3.4 Amplitude modulator using a diode.

Let the input-output characteristic of a diode be approximated by a power series :

$$v_o(t) = a v_i(t) + b v_i^2(t)$$

where  $a$  &  $b$  are constants and

$$v_i(t) = \cos 2\pi f_c t + m(t)$$

Hence :

$$\begin{aligned} v_o(t) &= a[\cos 2\pi f_c t + m(t)] + b[\cos 2\pi f_c t + m(t)]^2 \\ &= a m(t) + b \cos^2 2\pi f_c t + b m(t)^2 + \\ &\quad a \cos 2\pi f_c t + 2b m(t) \cos 2\pi f_c t \end{aligned}$$

If we pass the signal  $v_o(t)$  through a band- pass filter centered at  $\pm f_c$ , we obtain

$$\begin{aligned} v'_o(t) &= [a + 2b m(t)] \cos 2\pi f_c t \\ &= 2b[A + m(t)] \cos 2\pi f_c t \end{aligned}$$

where  $A = a / 2b$ . We generate a normal AM signal.

A normal amplitude-modulated signal can also be obtained by multiplying  $m(t)$  by a periodic signal  $s(t)$ . The modulator is called a *switching modulator*. If we take a periodic rectangular waveform  $s(t)$  of period  $T_c = 1/f_c$ , amplitude  $A_m$ , and pulse width  $\tau$ , the trigonometric Fourier series of  $s(t)$  is :

$$\begin{aligned} s(t) &= \frac{A_m \tau}{T_c} + \frac{2}{T_c} \sum_{n=1}^{\infty} \left( A_m \tau \frac{\sin 2\pi n f_c \tau / 2}{2\pi n f_c \tau / 2} \right) \cos 2\pi n f_c t \\ s(t) &= \frac{c_0}{T_c} + \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \end{aligned}$$

where  $c_0 = A_m \tau$  and

$$c_n = A_m \tau \frac{\sin 2\pi n f_c \tau / 2}{2\pi n f_c \tau / 2}$$

The corresponding complex Fourier series is :

$$s(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} \left( A_m \tau \frac{\sin 2\pi n f_c \tau / 2}{2\pi n f_c \tau / 2} \right) e^{j2\pi n f_c t}$$



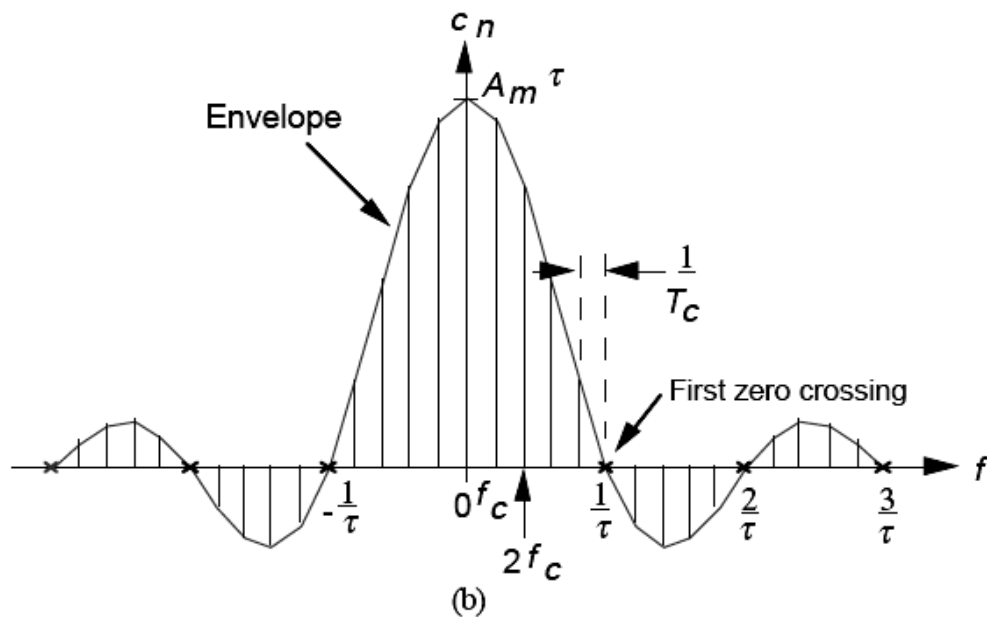
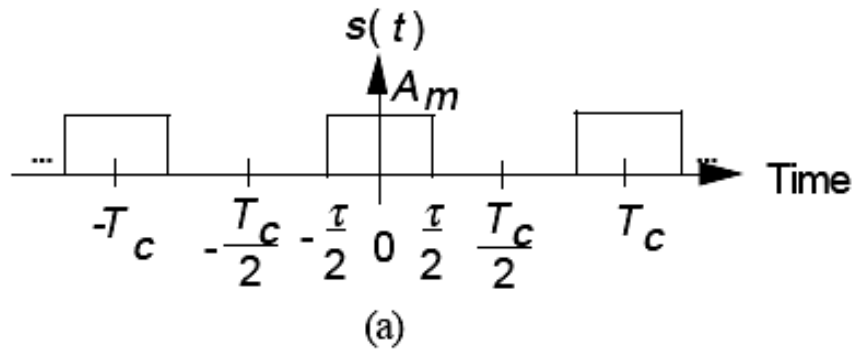


Figure 3.5 (a) A periodic rectangular waveform, and (b) its line spectrum.  
If the input signal is :

$$v_i(t) = \cos 2\pi f_c t + m(t),$$

the output of a switching modulator is :

$$\begin{aligned} v_o(t) &= v_i(t) s(t) \\ &= [\cos 2\pi f_c t + m(t)] s(t) \\ &= [\cos 2\pi f_c t + m(t)] \left( \frac{c_0}{T_c} + \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \right) \\ &= \left[ \frac{2}{T_c} \cos 2\pi f_c t \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \right] + \frac{c_0}{T_c} m(t) + \\ &\quad \frac{c_0}{T_c} \cos 2\pi f_c t + m(t) \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \end{aligned}$$

$v_o(t)$  consists of a dc term, the component  $m(t)$ , and an infinite number of normal AM signals at carrier frequencies  $f_c, 2f_c, 3f_c, \dots$ . If we pass the signal  $v_o(t)$  through a band-pass filter centered at  $\pm f_c$ , the filtered signal is :

$$v'_o(t) = \frac{c_0}{T_c} \cos 2\pi f_c t + \frac{2}{T_c} m(t)c_1 \cos 2\pi f_c t + \frac{c_2}{T_c} \cos 2\pi f_c t$$

$$= \frac{2c}{T_c} [A + m(t)] \cos 2\pi f_c t$$

where  $A = (c_0 + c_2) / 2c_1$ . That is, we obtain a normal AM signal.

### Demodulation of Normal AM Signals:

The process of recovering the message signal from the modulated signal is called **demodulation or detection**. Two basic methods are available :

#### 1. Envelope Detection :

In this method, an envelope detector is used to recover the message signal. An envelope detector consists of a diode and a resistor-capacitor combination. This is shown in Figure 3.6.

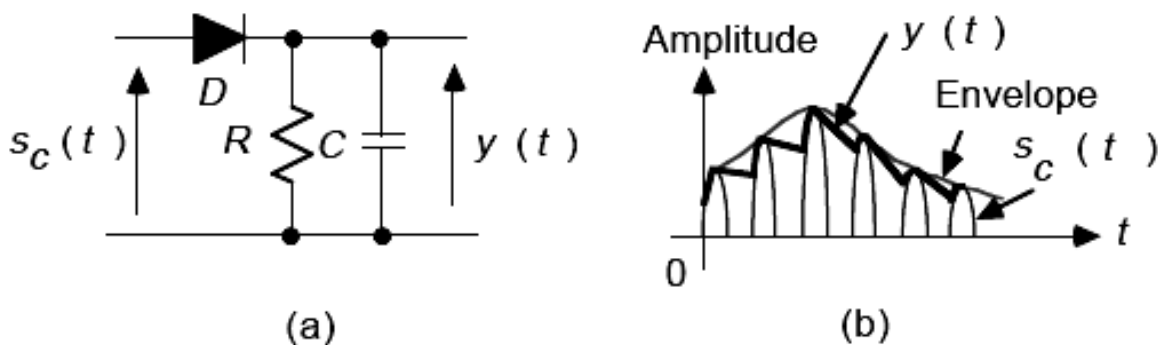
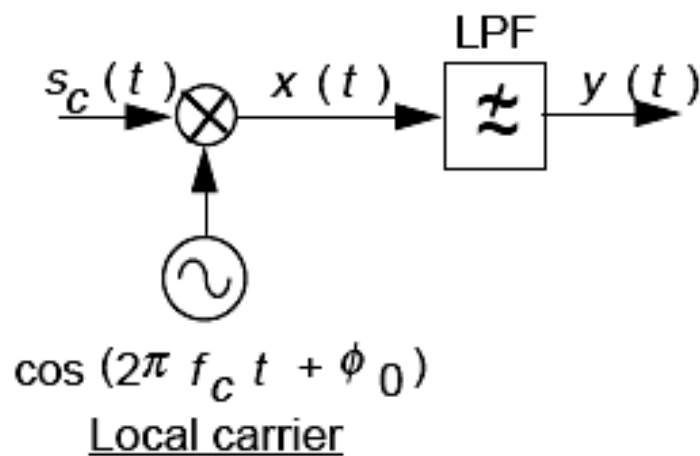


Figure 3.6 Envelope detector.

During the positive half-cycle peaks of the modulated signal, the diode is forward biased, and the capacitor charges up to the peak value of the modulated signal. As the modulated signal falls from its maximum, the diode turns off and the capacitor discharges through the resistor. The process repeats in this way. For proper operation, the discharge time constant  $RC$  must be chosen properly.

## 2.Synchronous (Coherent) Detection :

Here, a product detector is used to convert the band-pass signal to base-band. This is shown in Figure 3.7.



**Figure 3.7** Synchronous detector.

At the receiving end, the band-pass signal is multiplied by a locally generated carrier signal  $\cos(2\pi f_c t + \phi_0)$ , where  $\phi_0$  is an initial phase. The output of the multiplier is

$$\begin{aligned}
 x(t) &= [A + m(t)] \cos 2\pi f_c t \cos (2\pi f_c t + \phi_0) \\
 &= 0.5[A + m(t)] [\cos \phi_0 + \cos (4\pi f_c t + \phi_0)] \\
 &= 0.5[A + m(t)] \cos(4\pi f_c t + \phi_0) + 0.5A \cos \phi_0 + 0.5m(t) \cos \phi_0
 \end{aligned}$$

If we suppress the first term by a low-pass filter, we get

$$y(t) = 0.5A \cos \phi_0 + 0.5 m(t) \cos \phi_0$$

It can be seen that we can recover the component  $m(t)$  if the initial phase  $\phi_0$  is constant and small.

Suppose that the local carrier signal is :

$$\cos[2\pi(f_c + \Delta f)t],$$

The multiplier output becomes :

$$\begin{aligned} x(t) &= [A + m(t)] \cos 2\pi f_c t \cos[2\pi(f_c + \Delta f)t] \\ &= 0.5[A + m(t)] [\cos 2\pi\Delta f t + \cos 2\pi(2f_c + \Delta f)t] \\ &= 0.5[A + m(t)] \cos 2\pi(2f_c + \Delta f)t + \\ &\quad 0.5A \cos 2\pi\Delta f t + 0.5m(t) \cos 2\pi\Delta f t \end{aligned}$$

If we suppress the first term by a low-pass filter, we get

$$y(t) = 0.5A \cos 2\pi\Delta f t + 0.5m(t) \cos 2\pi\Delta f t$$

We cannot recover the component  $m(t)$  unless the frequency drift  $\Delta f$  is zero. Therefore, the local carrier must not only be of the same frequency but must be synchronized in phase with the carrier signal. If the carrier shifts in frequency or phase, the resultant signal is distorted or attenuated. Synchronous detection is sometimes called ***coherent detection***.

## Double-Sideband Modulation (DSB) :

A normal amplitude-modulated signal is given by :

$$s_c(t) = [A + m(t)] \cos 2\pi f_c t$$

For double-sideband (DSB) modulation,  $A = 0$  and

$$s_c(t) = m(t) \cos 2\pi f_c t$$

The Fourier transform of  $s_c(t)$  is

$$S_c(f) = 1/2 [M(f-f_c) + M(f+f_c)]$$

Figure 3.8 shows the waveforms and spectra associated with a DSB signal. Clearly, the envelope of the modulated signal does not have the same shape as  $m(t)$ . As with AM, DSB modulation shifts the spectrum of  $m(t)$  to the carrier frequency  $f_c$ . The bandwidth of the modulated signal is  $2 fm$  Hz, where  $fm$  is the bandwidth of the modulating signal  $m(t)$ .

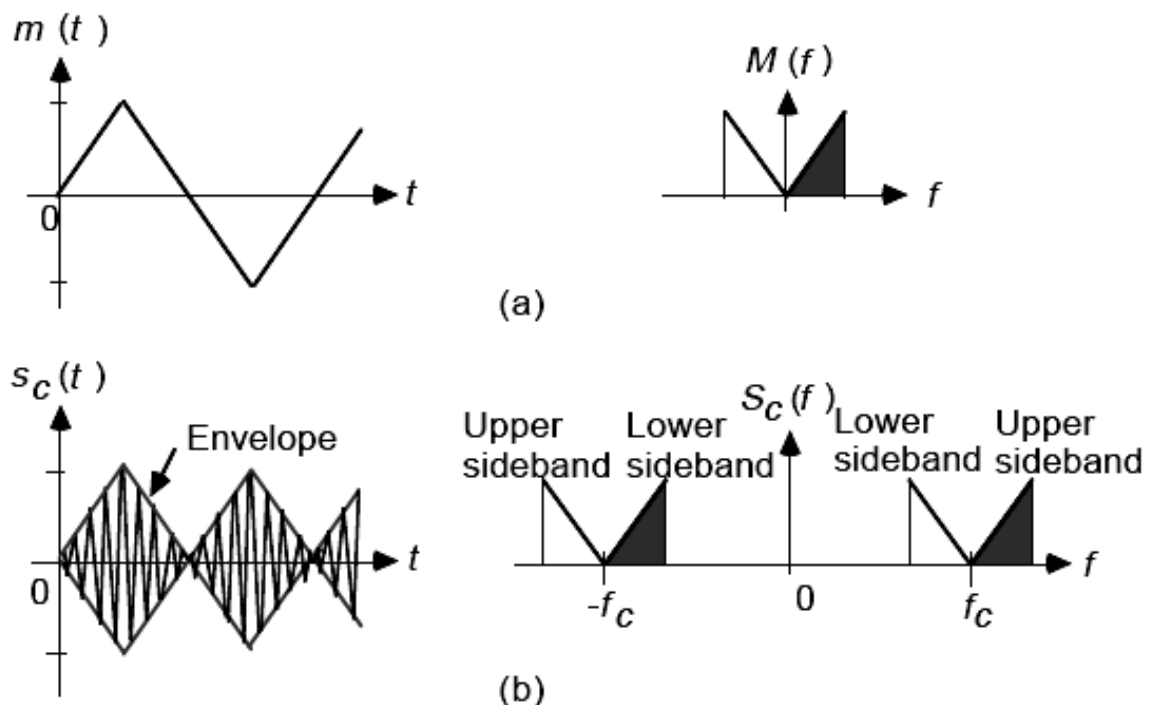


Figure 3.8 Waveforms and spectra associated with DSB signal.

## Generation of DSB Signals :

The process of generating a DSB signal is shown in Figure 3.9 .

DSB modulation can be achieved by using non-linear devices, such as a diode. This is shown in Figure 3.10 .

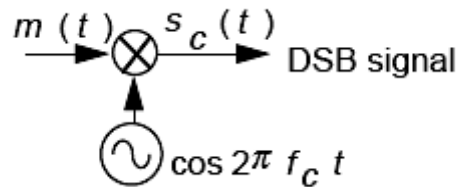


Figure 3.9 Generation of DSB signal.

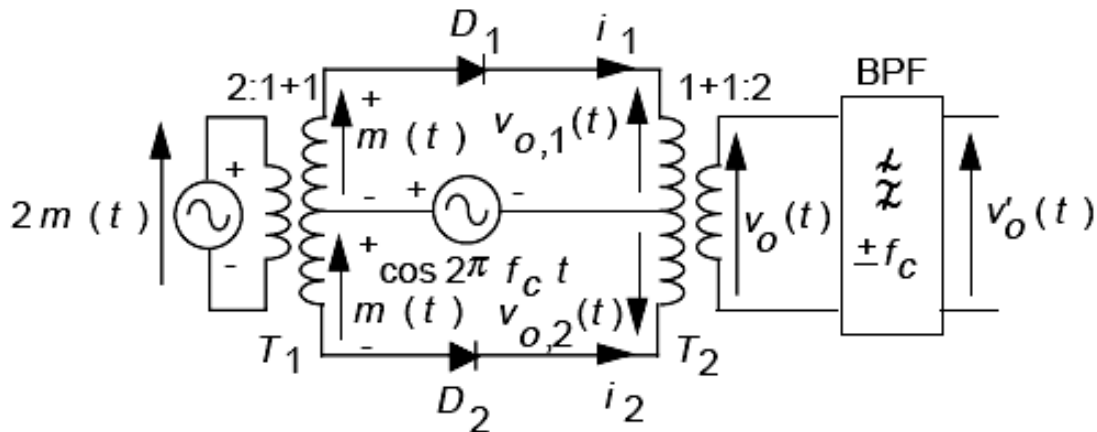


Figure 3.10 A single-balanced modulator.

Let the input-output characteristic of a diode be approximated by a power series :

$$v_o(t) = a v_i(t) + b v_i^2(t)$$

where  $a, b$  are constants. Consider the diode  $D_1$  in the upper portion of the circuit shown in Figure 3.10 . The input voltage to the diode  $D_1$  is

$$v_{i,1}(t) = v_i(t) = \cos 2\pi f t + m(t)$$

let the output voltage of the diode be

$v_{o,1}(t) = v_o(t)$ , then , we have :

$$\begin{aligned} v_{o,1}(t) &= a[\cos 2\pi f_c t + m(t)] + b[\cos 2\pi f_c t + m(t)]^2 \\ &= a m(t) + b \cos^2 2\pi f_c t + b m(t)^2 + \\ &\quad a \cos 2\pi f_c t + 2b m(t) \cos 2\pi f_c t \end{aligned}$$

Now, consider the diode  $D_2$  in the lower portion of the circuit shown in Figure(3.10). The input voltage to the diode  $D_2$  is :

$$v_{i,2}(t) = v_i(t) = \cos 2\pi f_c t - m(t)$$

and let the output voltage of the diode be

$v_{o,2}(t) = v_o(t)$ , then we have

$$\begin{aligned} v_{o,2}(t) &= a[\cos 2\pi f_c t - m(t)] + b[\cos 2\pi f_c t - m(t)]^2 \\ &= -a m(t) + b \cos^2 2\pi f_c t + b m(t)^2 + \\ &\quad a \cos 2\pi f_c t - 2b m(t) \cos 2\pi f_c t \end{aligned}$$

Hence ,

$$v_o(t) = v_{o,1}(t) - v_{o,2}(t) = 2a m(t) + 4b m(t) \cos 2\pi f_c t$$

If we pass this signal through a bandpass filter centred at  $\pm fc$ , we obtain

$$v'_o(t) = 4b m(t) \cos 2\pi f_c t$$

That is, we generate a DSB signal. It can be seen that the circuit is balanced with respect to the carrier signal  $s(t)$ ; however, the modulating signal  $m(t)$  still appears at the output of the second transformer  $T_2$ . For this reason, the modulator is called a *single-balanced modulator*.

From our earlier study of normal AM, multiplication of a signal by a periodic signal involves switching the signal  $m(t)$  on and off periodically. This can be accomplished by switching elements controlled by the carrier signal  $s(t)$ . Figure 3.11 shows a single balanced shunt-bridge diode modulator, where diodes  $D_1, D_2$  and  $D_3, D_4$  are matched pairs.

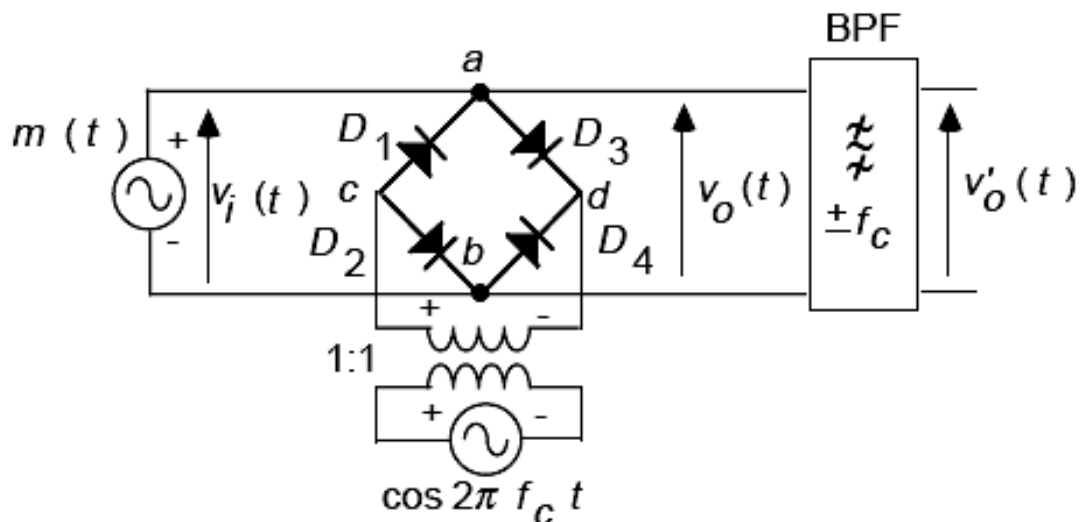


Figure 3.11 A single-balanced shunt-bridge diode modulator.

During the positive half-cycle of  $s(t)$ , terminal  $c$  is positive with respect to  $d$ , so all the diodes conduct. Terminals  $a$  and  $b$  have the same potential and are effectively shorted.  $v_o(t)$  is zero. During the negative half-cycle of  $s(t)$ , terminal  $c$  is negative with respect to  $d$ , all the diodes are open.

$v_o(t) = v_i(t) \cdot m(t)$ .  $m(t)$  is effectively multiplied by a non-negative periodic signal. Therefore, the component  $m(t)$  appears at the input to the bandpass filter. If we pass the signal  $v_o(t)$  through a bandpass filter centred at  $\pm f_c$ , we generate a DSB signal.

A DSB signal can also be obtained by multiplying  $m(t)$  by any periodic signal  $s(t)$ . The modulator is called a *switching modulator*.

If we take a periodic square waveform  $s(t)$  of period  $T_c = 1/f_c$ , amplitude  $\pm Ac/2$ , and pulse width of  $T_c$ , the trigonometric Fourier series of  $s(t)$  is :



$$s(t) = \frac{2}{T_c} \sum_{n=1}^{\infty} \left( 0.5A_c T_c \frac{\sin n\pi / 2}{n\pi / 2} \right) \cos 2\pi n f_c t$$

$$s(t) = \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t$$

where

$$c_n = 0.5A_c T_c \frac{\sin n\pi / 2}{n\pi / 2}$$

The corresponding complex Fourier series is:

$$s(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} \left( 0.5A_c T_c \frac{\sin n\pi / 2}{n\pi / 2} \right) e^{j2\pi n f_c t}$$

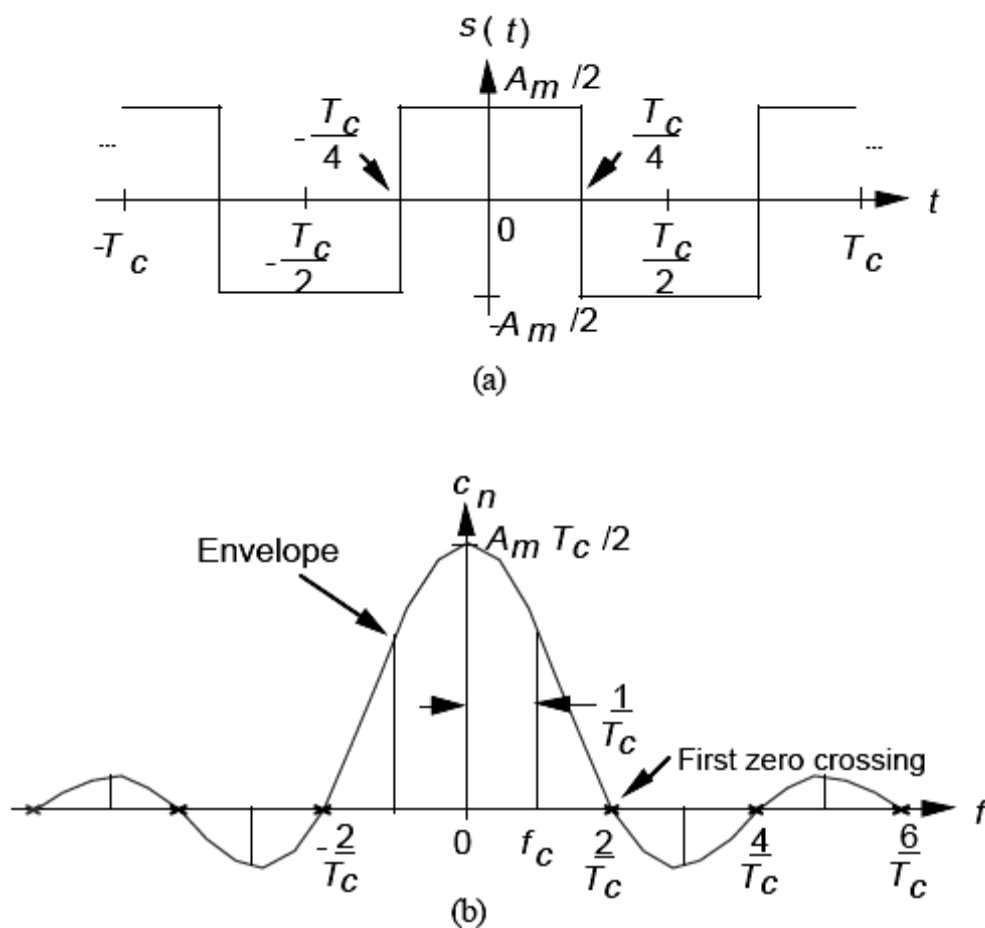


Figure 3.12 (a) A periodic square waveform, and (b) its line spectrum.

If the input signal is  $v_i(t) = m(t)$ , the product of  $v_i(t)$  and  $s(t)$  is :

$$\begin{aligned} v_o(t) &= v_i(t) s(t) \\ &= m(t) s(t) \\ &= m(t) \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \end{aligned}$$

$v_o(t)$  consists of an infinite number of DSB signals at carrier frequencies  $f_c, 2f_c, 3f_c, \dots$ . If we pass the signal  $v_o(t)$  through a bandpass filter centred at  $\pm f_c$ , the filtered signal is :

$$v'_o(t) = \frac{2}{T_c} c_1 m(t) \cos 2\pi f_c t$$

That is, we obtain a DSB signal. Figure 3.13 shows the waveforms and spectra associated with a switching modulator.

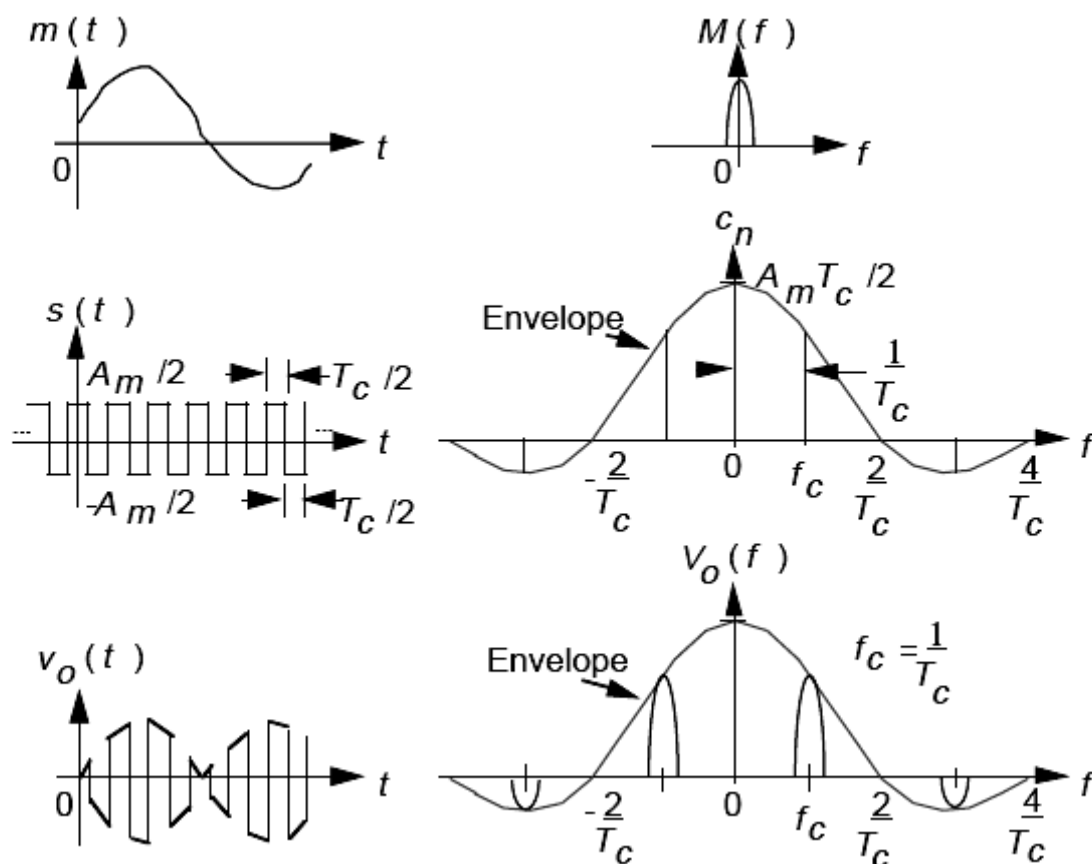


Figure 3.13 Waveforms and spectra associated with a switching modulator.

It is possible to design a balanced modulator such that the input to the bandpass filter does not contain the message signal  $m(t)$  or the carrier signal  $s(t)$ . A circuit balanced with respect to both input signals is called a *double-balanced modulator*. Figure 3.14 shows a double-balanced modulator, known as a *ring modulator*.

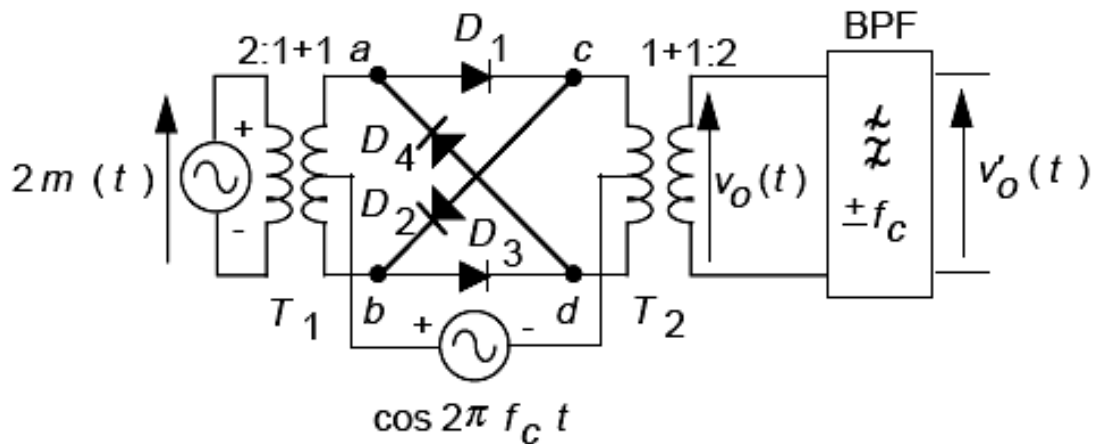


Figure 3.14 A double-balanced ring modulator.

During the positive half-cycle of  $s(t)$ , diodes  $D_1$  and  $D_3$  conduct, and  $D_2$  and  $D_4$  are open. Terminal  $a$  is connected to  $c$ , terminal  $b$  is connected to  $d$ , and  $v_o(t)$  is proportional to  $m(t)$ . During the negative half-cycle of  $s(t)$ , diodes  $D_1$  and  $D_3$  are open, and diodes  $D_2$  and  $D_4$  are conducting. Terminal  $a$  is connected to  $d$ , terminal  $b$  is connected to  $c$ , and  $v_o(t)$  is proportional to  $-m(t)$ .

$m(t)$  is effectively multiplied by a sinusoidal carrier waveform. Therefore, the components  $m(t)$  and  $s(t)$  do not appear at the input to the bandpass filter. If we pass the signal  $v_o(t)$  through a bandpass filter centred at  $\pm fc$ , we generate a DSB signal.

## Demodulation of DSB Signals :

Since the envelope of the modulated signal does not have the same shape as  $m(t)$ , an envelope detector cannot be used to recover the message signal. Demodulation of DSB signals can be accomplished by using a synchronous detector. This is shown in Figure 3.15 .

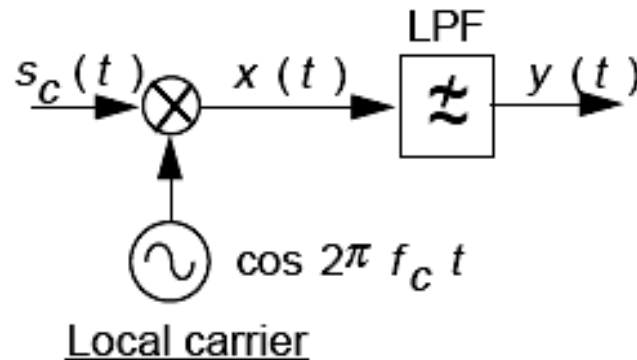


Figure 3.15 Synchronous detector.

Let  $s_c(t)$  be the input signal to the synchronous detector. At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal  $\cos 2\pi f_c t$ , which is in synchronism with the transmitted carrier signal. The output of the multiplier is :

$$\begin{aligned}x(t) &= m(t) \cos 2\pi f_c t \cos 2\pi f_c t \\ &= 0.5 m(t) + 0.5 m(t) \cos 4\pi f_c t\end{aligned}$$

If we suppress the last term by a low-pass filter, we get :

$$y(t) = 0.5m(t)$$

That is, we can recover the component  $m(t)$ . If the carrier signal shifts in frequency or phase, the resultant signal is distorted or attenuated.

For the demodulation of a DSB signal, we can use a squaring loop to generate a local carrier signal. Figure 3.16 shows a carrier-recovery squaring loop for a DSB signal .

However, the squaring loop has a disadvantage: there is a  $180^\circ$  phase ambiguity. If the transmitted carrier signal is shifted by  $180^\circ$ , the demodulated signal is  $-m(t)$ . The demodulated signal is inverted. To remove the phase ambiguity, we can send a known test signal or use *differential coding* and *decoding*.

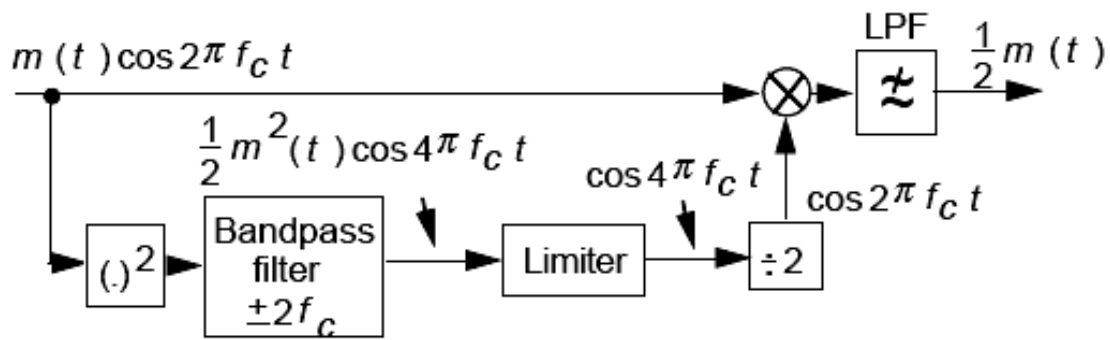


Figure 3.16 Carrier-recovery squaring loop for DSB signal.

Quadrature Amplitude Modulation (QAM) :

We have seen that DSB signals require a transmission bandwidth equal to twice the bandwidth of the message signal  $m(t)$ . To increase the transmission bandwidth efficiency, it is possible to send two DSB signals using carriers of the same frequency but in phase quadrature. Both modulated signals occupy the same frequency band. Yet they can be separated at the receiver by synchronous detection using two local carriers in phase quadrature. The technique is known as *Quadrature Amplitude Modulation (QAM)* or *quadrature multiplexing* and the arrangement is shown in Figure 3.17 .

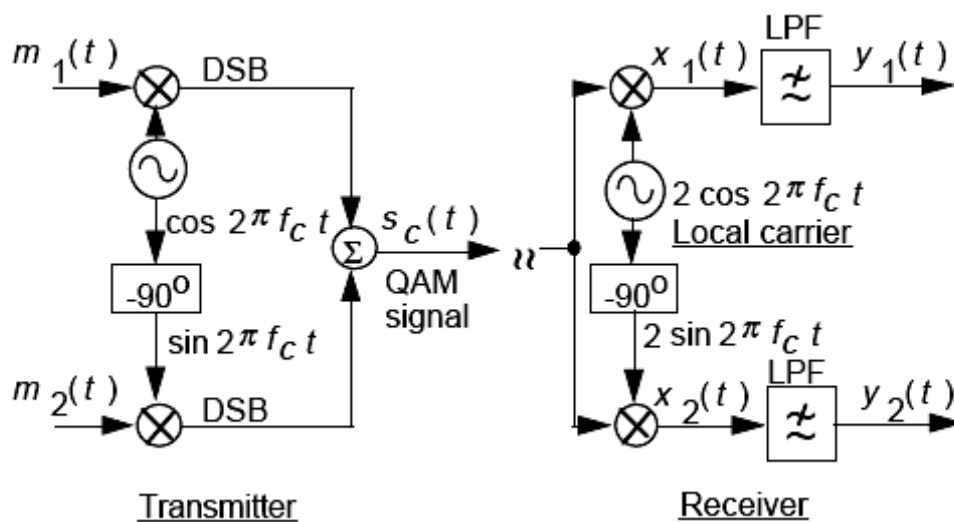


Figure 3.17 A QAM transmitter and receiver.

A QAM signal is given by :

$$s_c(t) = m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t$$

At the receiving end, the modulated signal is multiplied by two carriers in phase quadrature.

The signals at the outputs of the multipliers are :

$$\begin{aligned}x_1(t) &= 2 s_c(t) \cos 2\pi f_c t \\ &= m_1(t) + m_1(t) \cos 4\pi f_c t + m_2(t) \sin 4\pi f_c t\end{aligned}$$

and

$$\begin{aligned}x_2(t) &= 2 s_c(t) \sin 2\pi f_c t \\ &= m_2(t) - m_2(t) \sin 4\pi f_c t + m_1(t) \sin 4\pi f_c t\end{aligned}$$

If we suppress the high-frequency components by low-pass filters, we get

$$y_1(t) = m_1(t)$$

and

$$y_2(t) = m_2(t)$$

That is, the desired outputs are obtained. Supposing that the local carrier signal is  $\cos (2\pi f_c t + \phi_0)$ , then the multiplier output in the upper portion of the circuit becomes :

$$\begin{aligned}x_1(t) &= 2 s_c(t) \cos (2\pi f_c t + \phi_0) \\ &= m_1(t) \cos \phi_0 + m_1(t) \cos (4\pi f_c t + \phi_0) - \\ &\quad m_2(t) \sin \phi_0 + m_2(t) \sin (4\pi f_c t + \phi_0)\end{aligned}$$

If we suppress the second and the last terms by a low-pass filter, we get

$$y_1(t) = m_1(t) \cos \phi_0 + m_2(t) \sin \phi_0$$

The desired signal  $m_1(t)$  and the unwanted signal  $m_2(t)$  appear in the upper portion of the circuit. Also, it can be shown that  $y_2(t)$  contains the desired signal  $m_2(t)$  and the unwanted signal  $m_1(t)$ . Modulated signals having the same carrier frequency now interfere with each other. This is called ***cochannel interference*** and must be avoided .

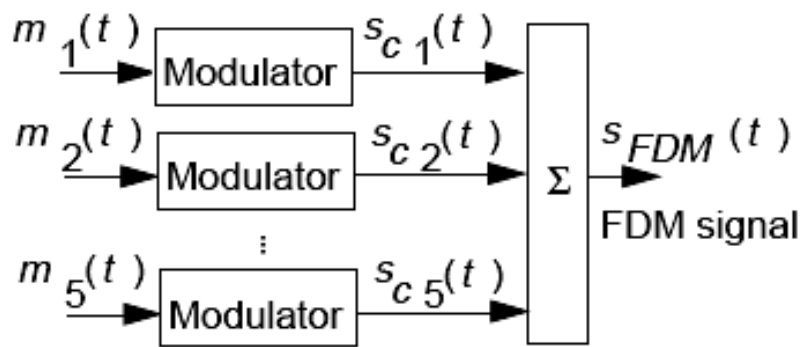
Worse problems arise when the local carrier frequency is in error. Therefore, the local carrier must not only be of the same frequency but must be synchronised in phase with the carrier signal. A slight error in the frequency or

the phase of the local carrier signal will not only result in loss and distortion of signals, but will also lead to interference.

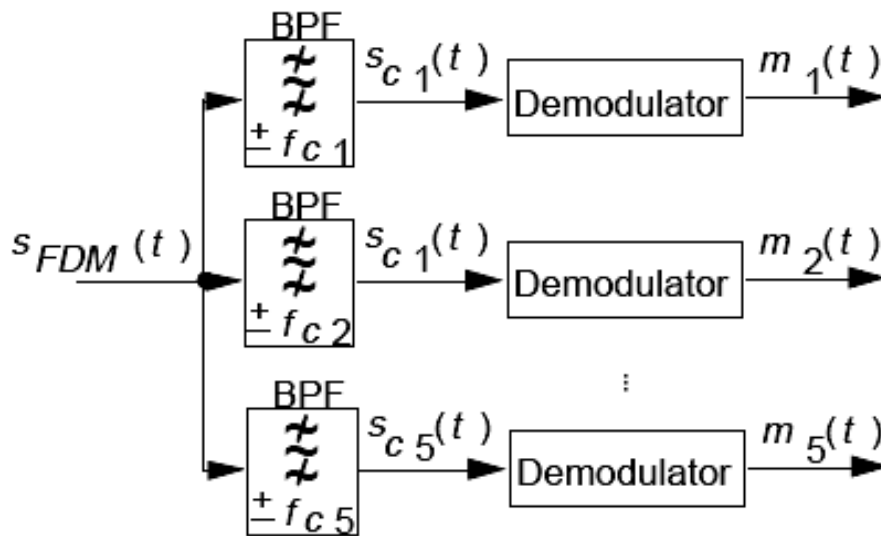
Quadrature multiplexing is used in colour television to multiplex the signals which carry the information about colours .

### Frequency Division Multiplexing (FDM) :

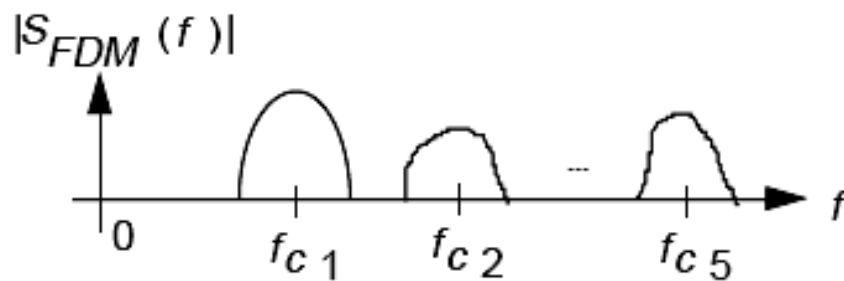
One of the basic problems in communication engineering is the design of a system which allows many individual signals from users to be transmitted simultaneously over a single communication channel. The most common method is to translate individual signals from one frequency region to another frequency region. Suppose that we have several different signals of the same bandwidth. If we translate each one of the signals to a different frequency region such that the translated signal spectra do not overlap each other, then all these signals can now be transmitted along a single communication channel. At the receiving end, the signals can be separated and recovered. We now have a frequency multiplexed system. Such a multiplexing technique is called *frequency division multiplexing (FDM)*. Frequency translation can be accomplished by multiplying a low-frequency modulating signal with a high-frequency sinusoidal carrier signal. Figure 3.18 shows the transmitter, the receiver, and the spectrum of a 5-user FDM system with carrier frequencies  $f_{c1} < f_{c2} < \dots < f_{c5}$ .



(a) Transmitter



(b) Receiver



(c) Spectrum of FDM signal

Figure 3.18 FDM system.



## Single-Sideband and Vestigial-Sideband Modulations :

We have seen that both normal AM and DSB signals require a transmission bandwidth equal to twice the bandwidth of the message signal  $m(t)$ . Since either the upper sideband (USB) or the lower sideband (LSB) contains the complete information of the message signal, we can conserve bandwidth by transmitting only one sideband. The modulation is called *single-sideband (SSB) modulation*.

### Generation of SSB Signals :

There are two common methods to generate a single-sideband signal.

#### 1- Filter Method :

In this method, a balanced modulator is used to generate a DSB signal, and the desired sideband signal is then selected by a bandpass filter for transmission. Figures 3.19 and 3.20 show the generation of a SSB signal using the filter method, and the spectra associated with it.

The technique is suitable for message signals with very little frequency content down to dc and hence does not require sharp filter cut-off characteristics.

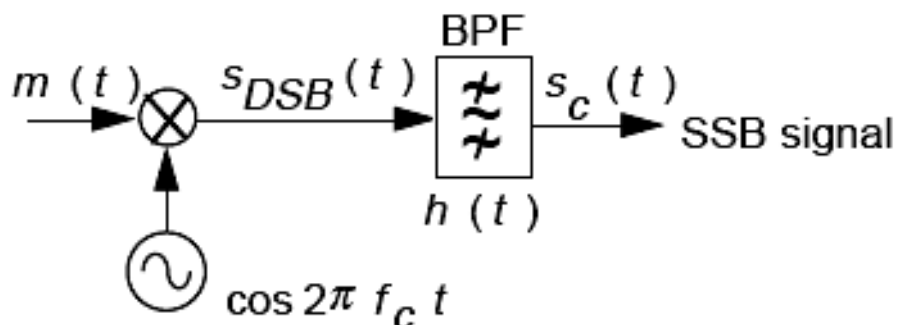


Figure 3.19 Generation of SSB signal using filter method.

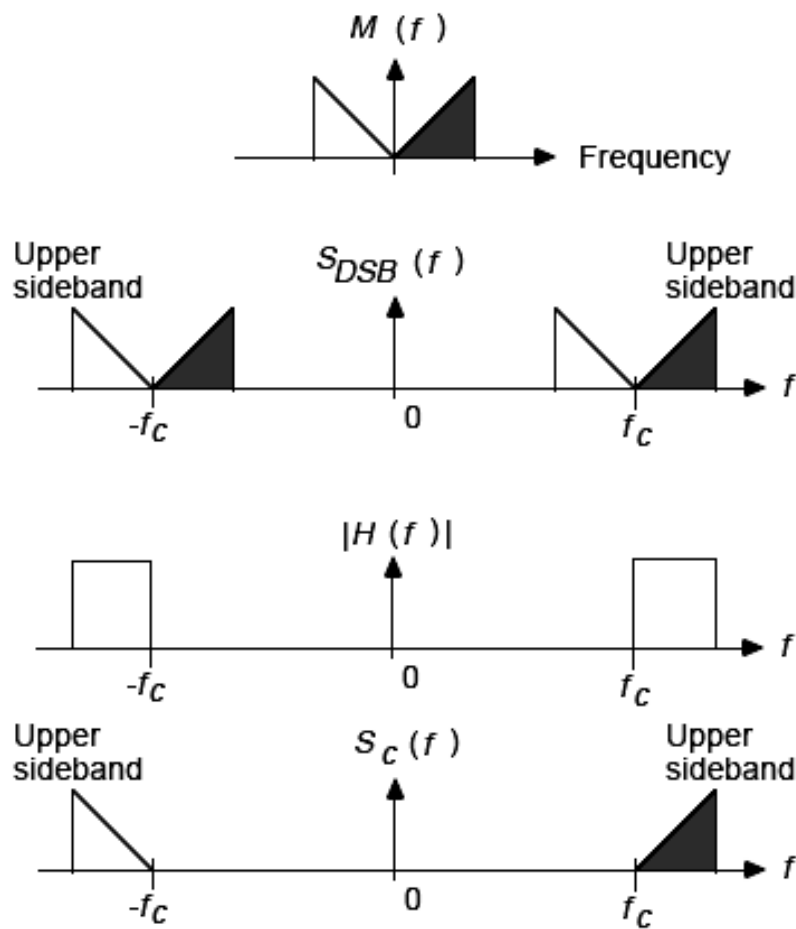


Figure 3.20 Spectra associated with SSB signal using filter method.

## 2- Phasing Method :

A single-sideband signal is given by :

$$s_c(t) = m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t$$

where the - sign is associated with the USB and the + sign is associated with the LSB.  $\hat{m}(t)$  is obtained by shifting the phase of all frequency components of  $m(t)$  by  $-90^\circ$ .

Figures 3.21 and 3.22 show the generation of a SSB signal using the phasing method, and the spectra associated with it.

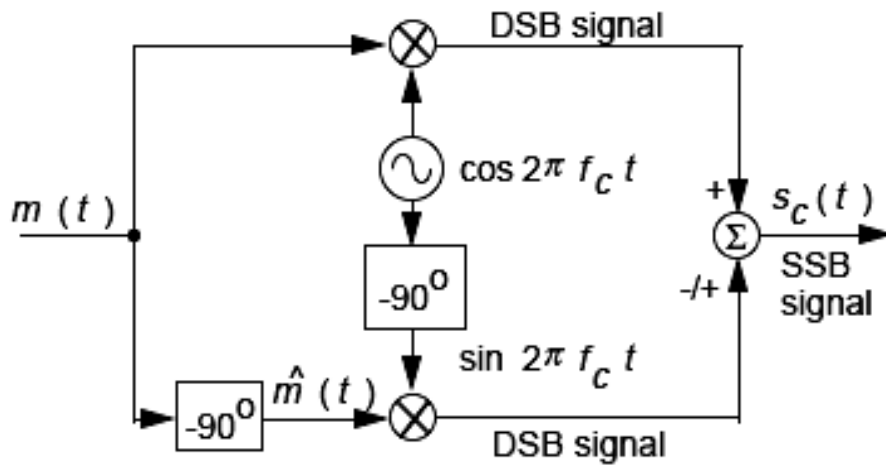


Figure 3.21 Generating SSB signal using phasing method.

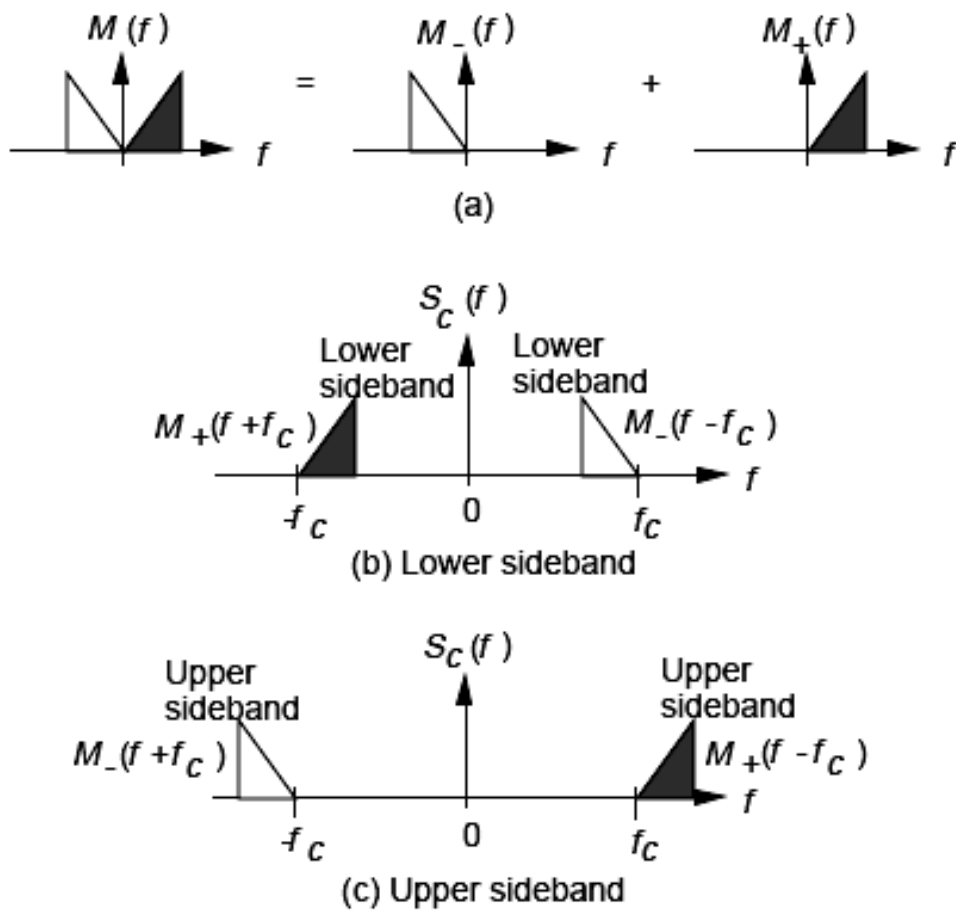


Figure 3.22 SSB spectra expressed in terms of  $M+(f)$  and  $M-(f)$ .

## Demodulation of SSB Signals :

Demodulation of SSB signals can be accomplished by using a synchronous detector as used in the demodulation of normal AM and DSB signals. This is shown in Figure 3.23 .

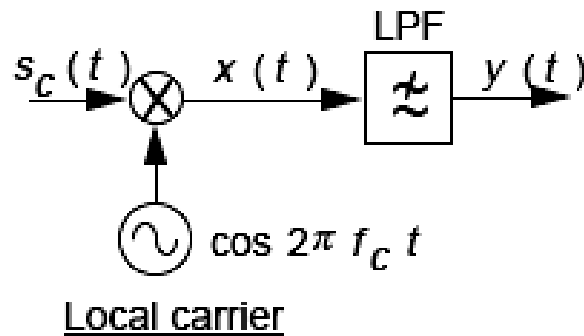


Figure 3.23 Synchronous detector.

At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal  $\cos 2\pi f_c t$ , which is in synchronism with the transmitted carrier signal. The output of the multiplier is :

$$\begin{aligned}x(t) &= [m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t] \cos 2\pi f_c t \\&= m(t) \cos^2 2\pi f_c t + [\hat{m}(t) \sin 2\pi f_c t] \cos 2\pi f_c t \\&= 0.5 m(t) + 0.5 m(t) \cos 4\pi f_c t + 0.5 \hat{m}(t) \sin 4\pi f_c t\end{aligned}$$

If we suppress the last two terms using a low-pass filter, we get :

$$y(t) = 0.5 m(t)$$

That is, we can recover the component  $m(t)$ . If the carrier signal has phase or frequency errors, the recovered message is distorted.

## Vestigial Sideband (VSB) Modulation :

Vestigial sideband modulation is a compromise between DSB and SSB modulations. It relaxes the sharp cutoff requirement of a SSB signal by retaining a trace of the other sideband in the transmitted signal. Typically, the bandwidth of a VSB modulated signal is about 1.25 times that of the corresponding SSB modulated signal. It is commonly used for transmission of video signals in commercial television broadcasting. Figures 3.24 and 3.25 show the generation of a VSB signal, and the spectra associated with it.

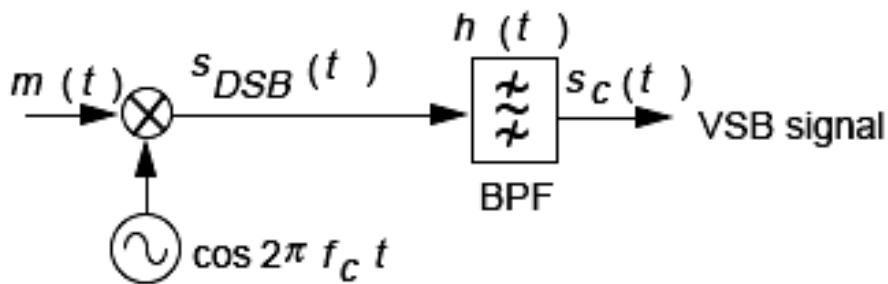


Figure 3.24 VSB modulator.

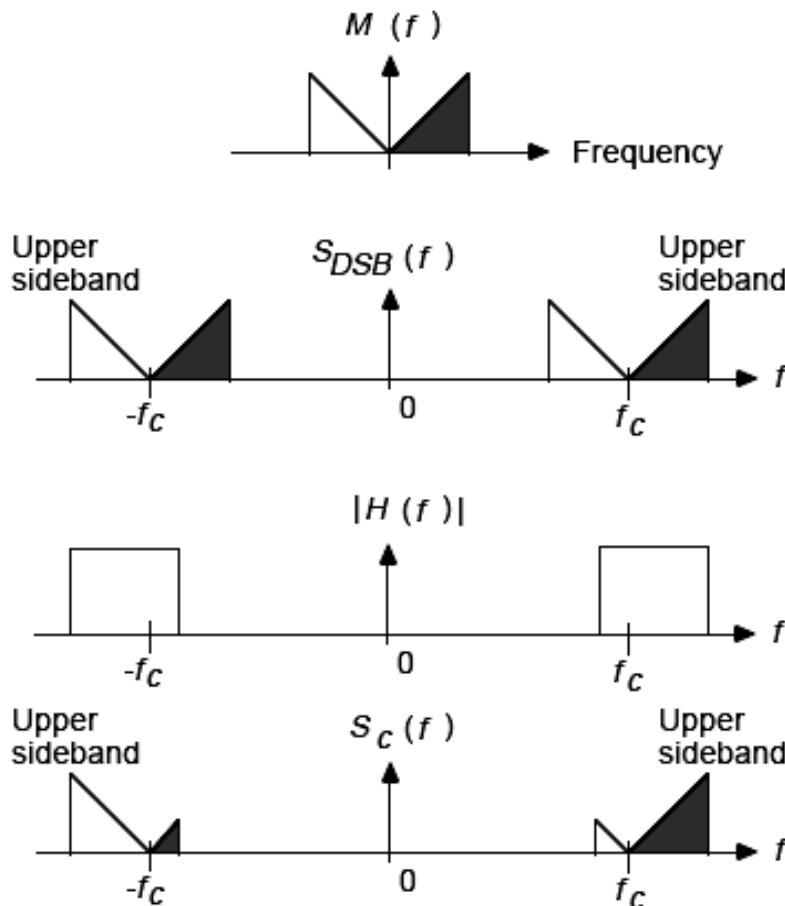


Figure 3.25 Spectra associated with VSB signal.

By inspection of Figure 3.25 , the Fourier transform of a VSB modulated signal  $s_c(t)$  is :

$$S_c(f) = 1/2 [M(f-f_c) + M(f+f_c)]H(f)$$

where  $H(f)$  is the transfer function of the bandpass filter.

### Demodulation of VSB Signals :

Demodulation of VSB signals can be accomplished by using a synchronous detector. Let  $s_c(t)$  be the input signal to the synchronous detector. At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal  $\cos 2\pi f_c t$ , which is in synchronism with the transmitted carrier signal. The output of the multiplier is :

$$x(t) = s_c(t) \cos 2\pi f_c t$$

and the Fourier transform of  $x(t)$  is :

$$X(f) = 1/2 [S_c(f-f_c) + S_c(f+f_c)]$$

Substituting , we get

$$X(f) = 1/2 \{ [ 1/2 [M(f-2f_c) + M(f)]H(f-f_c)] + [ 1/2 [M(f) + M(f+2f_c)]H(f+f_c)] \}$$

We can suppress the frequency components at  $\pm 2f_c$  by a low-pass filter and we get :

$$Y(f) = 1/4 M(f)[H(f-f_c) + H(f+f_c)]$$

For distortionless detection, we must have

$$H(f-f_c) + H(f+f_c) = K, \quad |f| \leq B$$

where  $K$  is a constant and  $B$  is the bandwidth of the message signal.

## The superheterodyne receiver :

Most modern radio and TV receivers (whether for AM or FM bands, other frequency ranges, or other modulation methods) use the *superheterodyne* (“*superhet*” for short) technique. This enables important signal processing operations, such as demodulation, to be done at a more convenient lower frequency, rather than at the original high frequency of the incoming signal, as well as providing other advantages.

A block diagram of a superheterodyne receiver for AM is shown in Fig. 3.26.

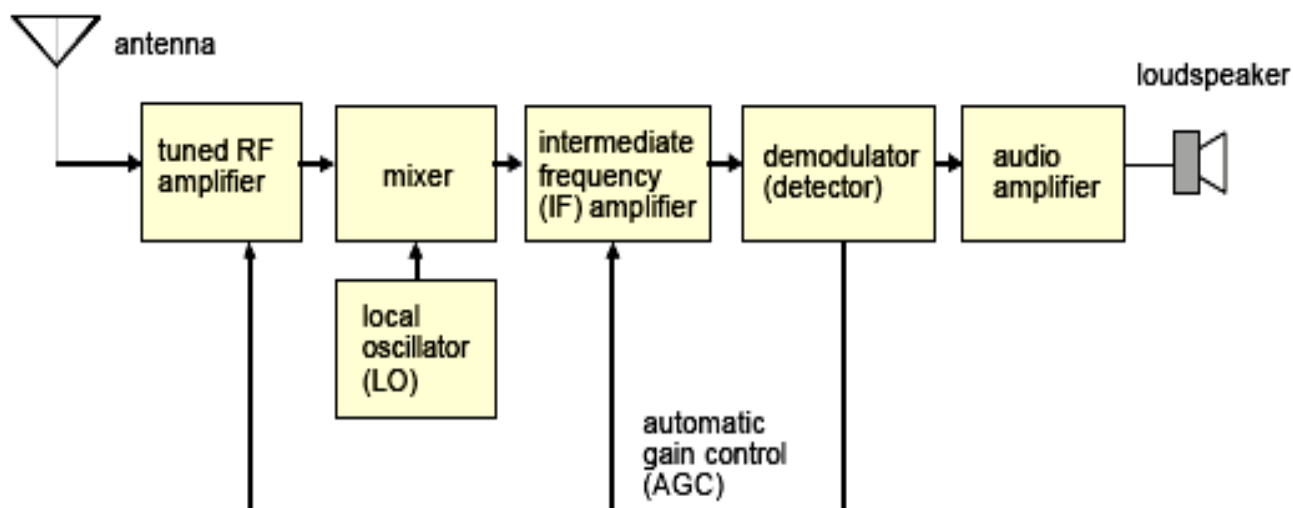


Fig 14-6 Diagram of a superheterodyne receiver for AM.

The signal is mixed with a local oscillator (LO) signal and shifted down to an intermediate frequency (IF) before demodulation.

- First, the received (modulated) signal is picked up by an antenna and passed through a *tuned amplifier* –that is, one which also acts as a *bandpass filter* as well as amplifying, to roughly restrict its frequency range.

This first part of the receiver is referred to as the *radio frequency (RF)* stage. The filtering has the effect of attenuating strong signals at other frequencies which might interfere with or overload the receiver’s amplifiers or mixer. To

select different stations, the tuning (that is, the centre frequency) of the bandpass filter is varied.

- The next stage is the *mixer*, where the signal is shifted down in frequency. In this stage, the signal from a separate oscillator (called the *local oscillator*, or *LO*) is multiplied by the incoming signal to produce sum and difference frequencies. The LO frequency is adjusted so that the difference frequency is always exactly 455 kHz. This is termed the *intermediate frequency (IF)* and is a standard frequency for AM receivers. (For AM, 470 kHz is also sometimes used; for FM receivers the IF is at 10.7 MHz, while in TV receivers it is around 30 MHz).

For example, if the incoming signal is at 1 MHz (1000 kHz), then an LO frequency of 1455 kHz (= 1000 + 455) or 545 kHz (= 1000-455) will produce an IF signal at 455 kHz; usually the higher LO frequency is used. As stations at different frequencies are selected by tuning the RF stage, the frequency of the LO is simultaneously varied, so as to keep the difference frequency at 455 kHz.

- This signal is then passed through an IF amplifier which is more precisely tuned to restrict the range of frequencies. Using a standard frequency for the IF means that no matter what the frequency of the original incoming (RF) signal, it is always processed in pretty much an identical fashion. Note that at this stage the signal still looks like an ordinary AM signal –that is, it has a carrier (but now at 455 kHz) and two sidebands.
- The amplified IF signal is then passed to the demodulator (detector), which recovers the original audio information from the carrier.
- An additional dc voltage derived from the detector is also used to provide a feedback voltage for automatic gain control, which operates by varying the gain of the IF and/or RF amplifiers, thus keeping the average signal level to the demodulator approximately constant.
- Finally, an audio amplifier raises the power of the recovered signal to a sufficient level to drive a loudspeaker.



**Example 3.1:**

The AM broadcast band occupies approximately 500 to 1600 kHz. An AM receiver uses an IF frequency of 455 kHz. It is tuned to a station, and its LO frequency is set to 1365 kHz. On what frequency is the station broadcasting?

**Answer:**

The IF must be at the sum or difference of the RF and LO frequencies.

Hence, the station frequency must be at either

$$1365 + 455 = 1820 \text{ kHz, or}$$

$$1365 - 455 = 910 \text{ kHz,}$$

in order to give an IF of 455 kHz. Since 1820 kHz is not in the AM band, the station frequency must be 910 kHz.

**Example 3.2:**

An AM receiver uses an IF frequency of 455 kHz. A radio station at 1200 kHz modulates its carrier with a maximum audio frequency of 9 kHz.

- (a) What are the highest and lowest frequency components that are actually broadcast by the station?
- (b) What are the highest and lowest frequencies that must be passed without much loss by the receiver's IF amplifier to recover the original audio signal with full bandwidth?

**Answer:**

- (a) The highest and lowest frequencies broadcast will correspond to the "outside edges" of the two AM sidebands. These will be at :  
 $1200 - 9 = 1191 \text{ kHz}$     and  
 $1200 + 9 = 1209 \text{ kHz}.$
- (b) The original carrier frequency will be shifted down to the IF frequency of 455 kHz by the mixer. Hence the highest and lowest corresponding frequencies which must be passed by the IF amplifier are:  
 $455 - 9 = 446 \text{ kHz}$     and  
 $455 + 9 = 464 \text{ kHz}.$

## 4. Angle Modulation

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM). The phase angle of a sinusoidal carrier signal is varied according to the modulating signal. In angle modulation, the spectral components of the modulated signal are not related in a simple fashion to the spectrum of the modulating signal. Superposition does not apply and the bandwidth of the modulated signal is usually much greater than the modulating signal bandwidth.

### Definitions

A bandpass signal is represented by

$$s_c(t) = A(t) \cos \theta(t)$$

where  $A(t)$  is the envelope and  $\theta(t) = \omega_c t + \phi(t) = 2\pi f_c t + \phi(t)$ .

For angle modulation, we can write :

$$s_c(t) = A \cos [2\pi f_c t + \phi(t)]$$

where  $A$  is a constant and  $\phi(t)$  is a function of the modulating signal.

$\phi(t)$  is called the instantaneous phase deviation of  $s_c(t)$ .

The instantaneous angular frequency of  $s_c(t)$  is defined as :

$$\omega_i(t) = d\theta(t) / dt$$

In terms of frequency, the instantaneous frequency of  $s_c(t)$  is :

$$\begin{aligned} f_i &= \frac{1}{2\pi} \frac{d\theta(t)}{dt} \\ &= f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \end{aligned}$$

$\frac{1}{2\pi} \frac{d\phi(t)}{dt}$  is known as the instantaneous frequency deviation.

The peak (maximum) frequency deviation is :

$$\Delta f = \max \left| \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right| = \max |f_i(t) - f_c|$$

## Phase Modulation :

For PM, the instantaneous phase deviation is proportional to the modulating signal  $m_p(t)$ :

$$\phi(t) = k_p m_p(t)$$

where  $k_p$  is a constant. Thus, a phase-modulated signal is represented by

$$s_c(t) = A \cos [2\pi f_c t + k_p m_p(t)]$$

Then, the instantaneous frequency of  $s_c(t)$  can be written as :

$$f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{dm_p(t)}{dt}$$

The peak (maximum) phase deviation is :

$$\begin{aligned} \Delta\phi &= \max|\phi(t)| \\ &= k_p \max|m_p(t)| \end{aligned}$$

The phase modulation index is given by :

$$\beta_p = \Delta\phi$$

## Frequency Modulation :

For FM, the instantaneous frequency deviation is proportional to the modulating signal  $m_f(t)$  :

$$\frac{d\phi(t)}{dt} = k_f m_f(t)$$

where  $k_f$  is a constant, and

$$\phi(t) = k_f \int_{-\infty}^t m_f(\tau) d\tau + \phi(-\infty)$$

$\phi(-\infty)$  is usually set to 0. Thus, a frequency-modulated signal is represented by:

$$s_c(t) = A \cos \left[ 2\pi f_c t + k_f \int_{-\infty}^t m_f(\tau) d\tau \right]$$

The instantaneous frequency of  $s_c(t)$  can be written as :

$$f_i(t) = f_c + \frac{1}{2\pi} k_f m_f(t)$$

Figure 4.1 shows the modulating signal  $m_f(t)$ , the instantaneous frequency  $f_i(t)$ , and the FM signal when a sawtooth signal is used as a modulating signal.

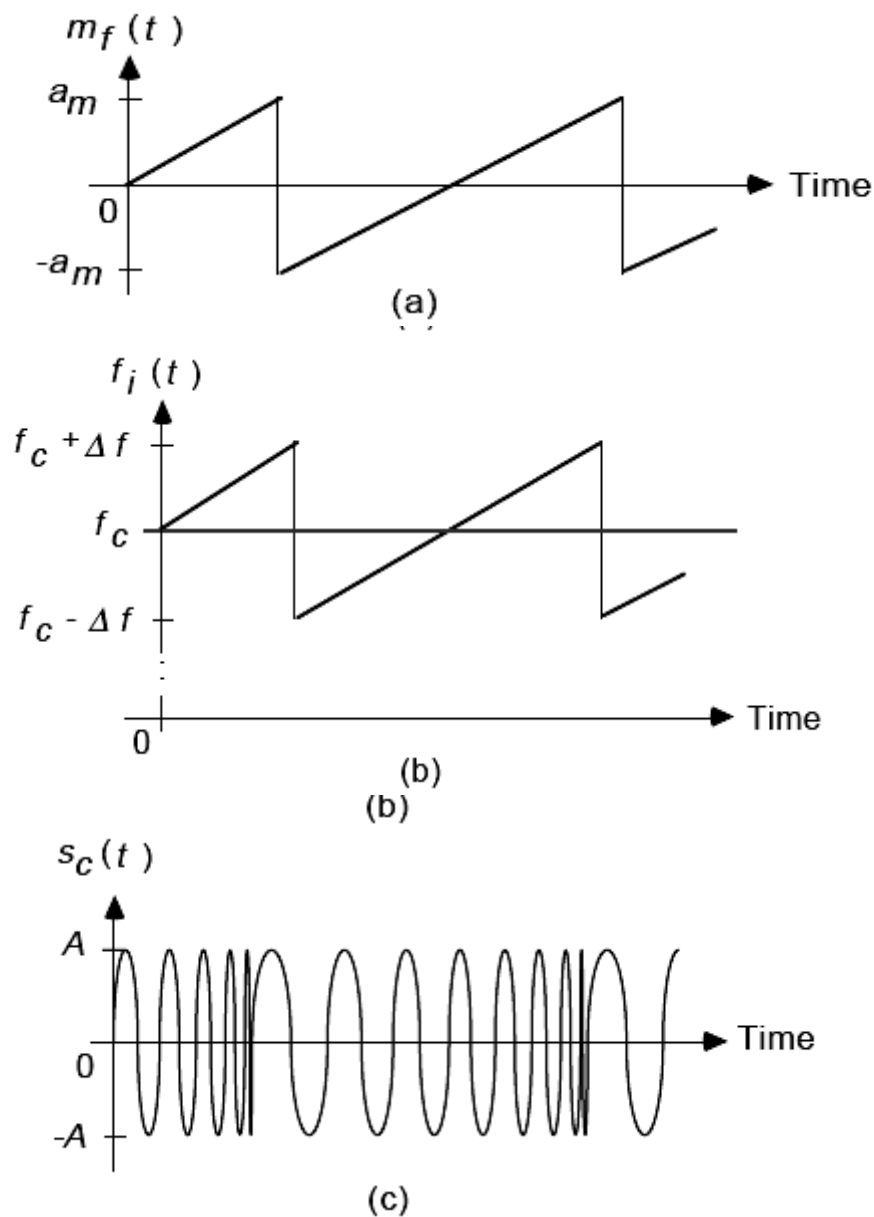


Figure 4.1 Frequency modulation: (a) Modulating signal, (b) instantaneous frequency, and (c) FM signal.

The frequency deviation from the carrier frequency is:

$$f_d(t) = f_i(t) - f_c = \frac{1}{2\pi} \frac{d\phi(t)}{dt} = \frac{1}{2\pi} k_f m_f(t)$$

and the peak frequency deviation is :

$$\begin{aligned} \Delta f &= \max \left| \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right| \\ &= \frac{1}{2\pi} k_f \max |m_f(t)| \end{aligned}$$

### **Generation of Angle-Modulated Signal :**

It can be seen from the previous equations that PM and FM differ only by a possible integration or differentiation of the modulating signal.

Then we obtain :

$$m_f(t) = \frac{k_p}{k_f} \frac{dm_p(t)}{dt}$$

and

$$m_p(t) = \frac{k_f}{k_p} \int_{-\infty}^t m_f(\tau) d\tau$$

If we differentiate the modulating signal  $m_p(t)$  and frequency-modulate using the differentiated signal, we get a PM signal. On the other hand, if we integrate the modulating signal  $m_f(t)$  and phase-modulate using the integrated signal, we get an FM signal.

Therefore, we can generate a PM signal using a frequency modulator or we can generate an FM signal using a PM modulator. This is shown in Figure 4.2.

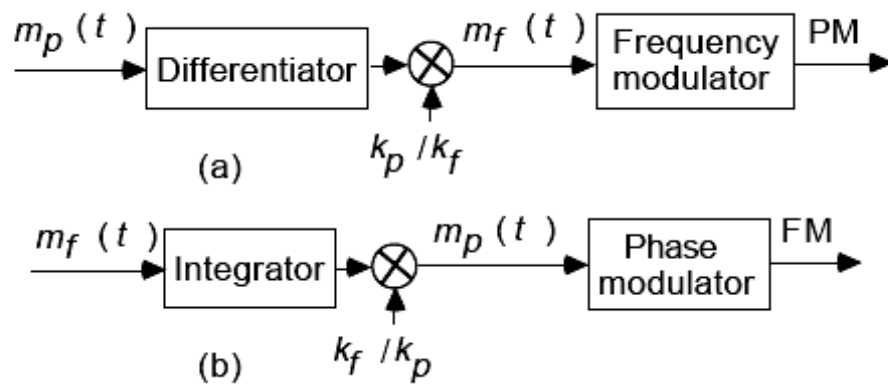


Figure 4.2 Generation of (a) PM using a frequency modulator, and (b) FM using a phase modulator.

### Spectrum of an Angle-Modulated Signal :

For angle modulation,

$$s_c(t) = A \cos [2\pi f_c t + \phi(t)]$$

and we can write :

$$s_c(t) = \text{Re} \{ A e^{j[2\pi f_c t + \phi(t)]} \} = \text{Re} \{ A e^{j2\pi f_c t} e^{j\phi(t)} \}$$

Expanding  $e^{j\phi(t)}$  in a power series yields :

$$s_c(t) = \text{Re} \left\{ A e^{j2\pi f_c t} \left[ 1 + j\phi(t) - \frac{\phi^2(t)}{2!} - \dots + jn \frac{\phi^n(t)}{n!} + \dots \right] \right\}$$

$$= A \left[ \cos 2\pi f_c t - \phi(t) \sin 2\pi f_c t - \frac{\phi^2(t)}{2!} \cos 2\pi f_c t + \frac{\phi^3(t)}{3!} \sin 2\pi f_c t + \dots \right]$$

It can be seen that the spectrum of an angle-modulated signal consists of an unmodulated carrier plus spectra of  $\phi(t)$ ,  $\phi^2(t)$ , ..., and is not related to the spectrum of the modulating signal in a simple fashion.

## Narrowband Angle Modulation :

If  $\max |\phi(t)| \ll 1$ , we can neglect all higher-power terms of  $\phi(t)$  in the last equation and we have a narrowband angle-modulated signal :

$$s_c(t) \approx A[\cos 2\pi f_c t - \phi(t)\sin 2\pi f_c t]$$

For PM,

$$s_c(t) \approx A[\cos 2\pi f_c t - k_p m_p(t)\sin 2\pi f_c t] \dots\dots\dots(4-1)$$

For FM,

$$s_c(t) \approx A \left\{ \cos 2\pi f_c t - \left[ k_f \int_{-\infty}^t m_f(\tau) d\tau \right] \sin 2\pi f_c t \right\} \dots\dots(4-2)$$

## Narrowband Frequency Modulation :

Because of the difficulty of analyzing general angle-modulated signals, we shall only consider a sinusoidal modulating signal. Let the modulating signal of a narrowband FM signal be :

$$m_f(t) = a_m \cos 2\pi f_m t$$

we have,

$$\begin{aligned} \phi(t) &= \frac{k_f a_m}{2\pi f_m} \sin 2\pi f_m t \\ &= \beta_f \sin 2\pi f_m t \end{aligned}$$

where ,  $\beta_f = k_f a_m / (2\pi f_m)$ .

$\beta_f$  is called the *frequency modulation index* and  $\beta_f$  is only defined for a sinusoidal modulating signal.

Differentiating  $\phi(t)$  and substituting  $d\phi(t)/dt$  into  $\Delta f$ , we have :

$$\beta_f = \frac{\Delta f}{B}$$

where  $B = f_m$  is the bandwidth of the modulating signal. Then, we have :

$$\begin{aligned} s_c(t) &= A \cos(2\pi f_c t + \beta_f \sin 2\pi f_m t) \\ &= A[\cos 2\pi f_c t \cos(\beta_f \sin 2\pi f_m t) - \sin 2\pi f_c t \sin(\beta_f \sin 2\pi f_m t)] \end{aligned}$$

For  $\beta_f \ll \pi/2$ ,

$\cos(\beta_f \sin 2\pi f_m t) \approx 1$ ,  $\sin(\beta_f \sin 2\pi f_m t) \approx \beta_f \sin 2\pi f_m t$ , and

$$\begin{aligned} s_c(t) &\approx A[\cos 2\pi f_c t - \beta_f \sin 2\pi f_c t \sin 2\pi f_m t] \\ &\approx A \cos 2\pi f_c t - 0.5\beta_f A[\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t] \\ &\approx \text{Re}[e^{j2\pi f_c t}(A - 0.5\beta_f A e^{-j2\pi f_m t} + 0.5\beta_f A e^{j2\pi f_m t})] \end{aligned}$$

The bandwidth of the narrowband FM signal is  $2f_m$  Hz.

In the AM case with sinusoidal modulating signal :

$$m(t) = a_m \cos 2\pi f_m t,$$

$$s_c(t) = [A + a_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$s_c(t) = A \cos 2\pi f_c t + 0.5a_m[\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t]$$

$$s_c(t) = A \cos 2\pi f_c t + 0.5mA[\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t]$$

$$s_c(t) = \text{Re}[e^{j2\pi f_c t}(A + 0.5mA e^{-j2\pi f_m t} + 0.5mA e^{j2\pi f_m t})]$$

where the modulation index :

$$m = \frac{a_m}{A}$$



Figure 4.3 shows the vector representation of a narrowband FM signal and an AM signal.

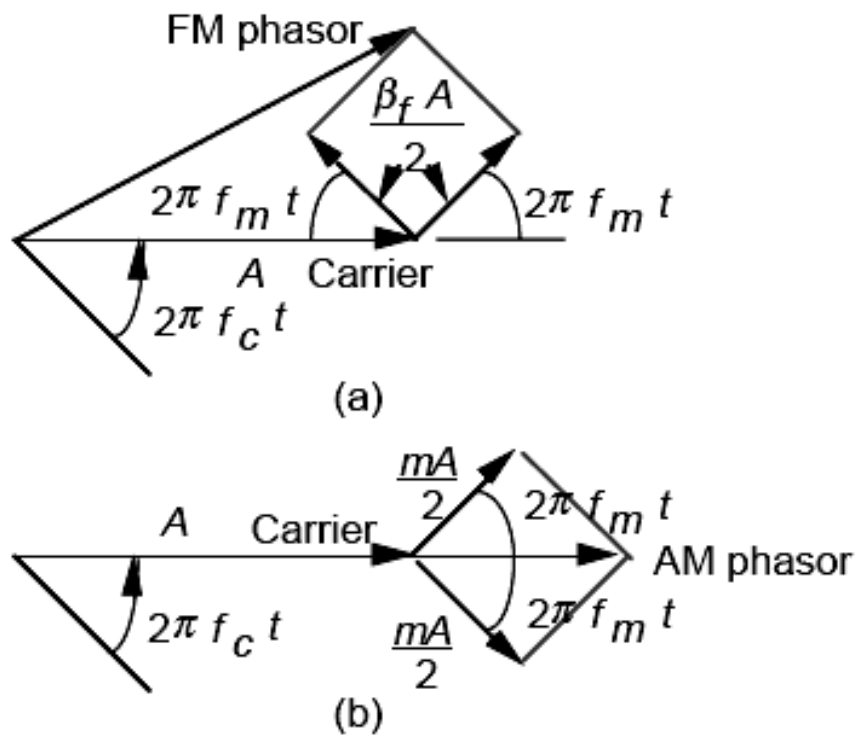


Figure 4.3 Vector representation of (a) narrowband FM, and (b) AM.

It can be seen that the resultant of the two sideband vectors in the FM case is always in phase quadrature with the unmodulated carrier, whereas the resultant of the two sideband vectors in the AM case is always in phase with the unmodulated carrier. The distinction and similarity between narrowband FM (or phase modulation) leads us to a commonly used method of generating narrowband angle-modulated signals.

## Generation of Narrowband PM and Narrowband FM :

The generation of narrowband PM and narrowband FM signals is easily accomplished in view of equations (4-1) & (4-2) . This is shown in Figure 4.4.

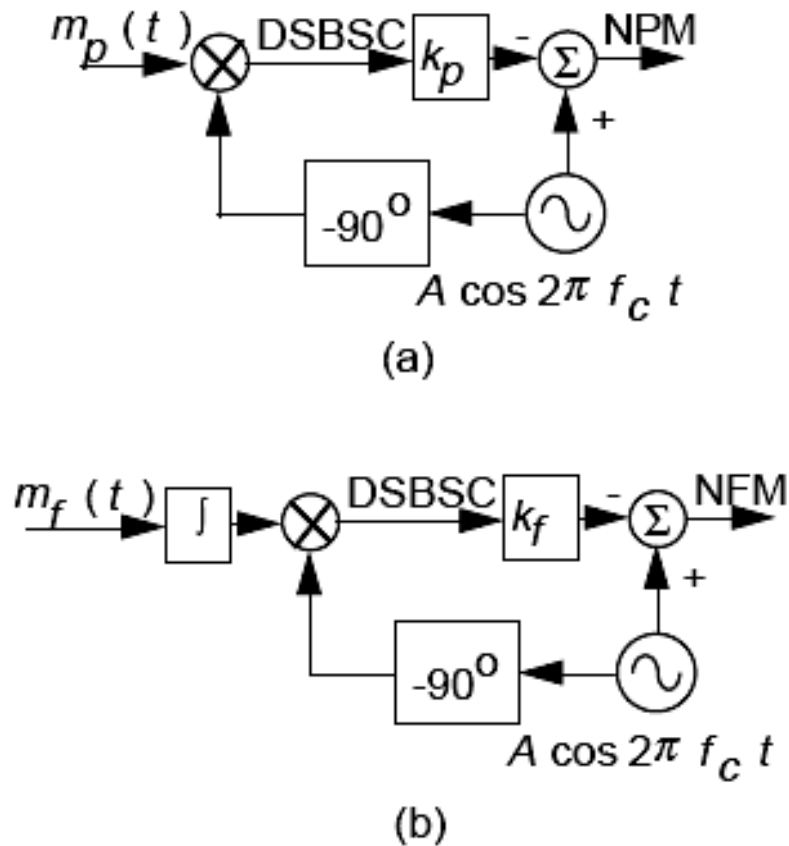


Figure 4.4 Generation of (a) narrowband PM, and (b) narrowband FM.

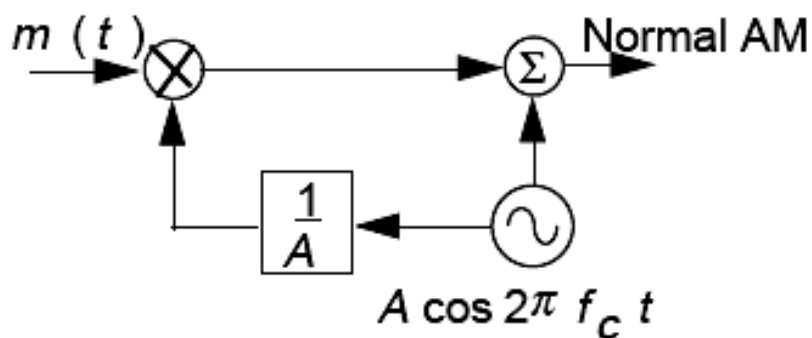


Figure 3.3 Generation of normal AM signal.

## Wideband Frequency Modulation (WFM) :

Consider the angle-modulated signal :

$$s_c(t) = A \cos (2\pi f_c t + \beta \sin 2\pi f_m t)$$

with sinusoidal modulating signal

$$m(t) = a_m \cos 2\pi f_m t.$$

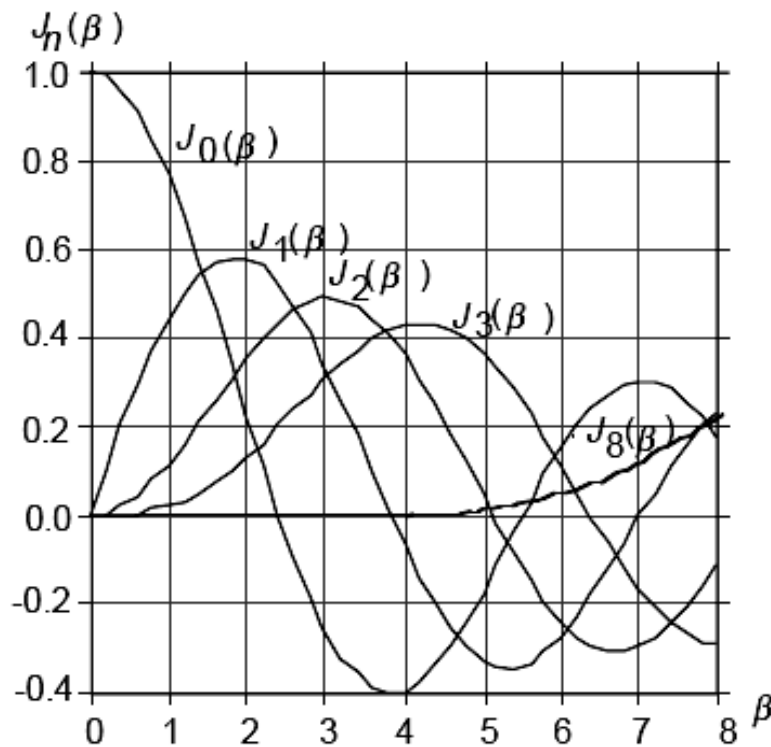
It can be shown that  $s_c(t)$  can also be written as :

$$s_c(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t$$

where :

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

The integral is known as the Bessel function of the first kind of the n-th order and cannot be evaluated in closed form. Figure 4.5 shows some Bessel functions for  $n = 0, 1, 2, 3,$  and  $8$ . Clearly, the value of  $J_n(\beta)$  becomes small for large values of  $n$ .



**Figure 4.5** Bessel functions.

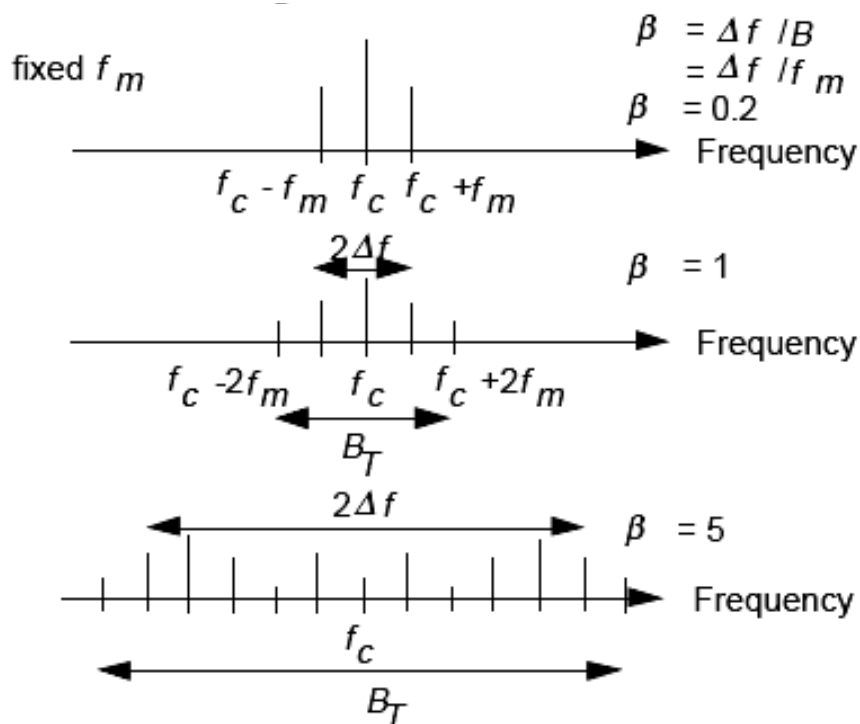
It can be shown that :

$$J_n(\beta) = \begin{cases} J_{-n}(\beta), \dots n..even \\ -J_{-n}(\beta), \dots n..odd \end{cases}$$

Therefore, we can write :

$$s_c(t) = A \{ J_0(\beta) \cos 2\pi f_c t - J_1(\beta) [\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t] + J_2(\beta) [\cos 2\pi(f_c - 2f_m)t + \cos 2\pi(f_c + 2f_m)t] - J_3(\beta) [\cos 2\pi(f_c - 3f_m)t - \cos 2\pi(f_c + 3f_m)t] + \dots \} \dots (4-3)$$

Figure 4.6 shows the amplitude spectra of FM signals with a sinusoidal modulating signal and fixed  $f_m$ .

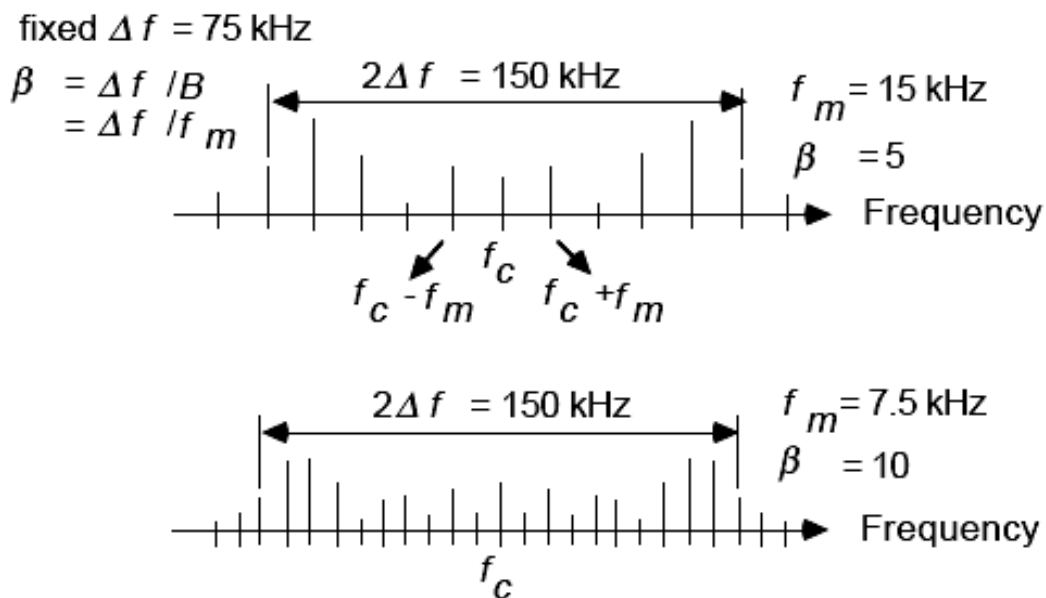


**Figure 4.6** Amplitude spectra of FM signals with sinusoidal modulating signal and fixed  $f_m$ .

## Observations:

1. The spectrum consists of a carrier component at  $f_c$  plus sideband components at  $f_c \pm n f_m$  ( $n = 1, 2, \dots$ ).
2. The number of sideband terms depends on the modulation index  $\beta$ .
3. The magnitude of the carrier signal decreases rapidly as  $\beta$  increases.
4. The amplitudes of the spectral lines depend on the value of  $J_n(\beta)$ .
5. The bandwidth of the modulated signal with a sinusoidal modulating signal increases as  $\beta$  increases, and the bandwidth of the modulated signal is larger than  $2\Delta f$ .

Figure 4.7 shows the amplitude spectra of FM signals with a sinusoidal modulating signal and a fixed peak frequency deviation  $\Delta f$ . Clearly, we get more and more spectral lines crowding into a fixed frequency interval as  $f_m$  decreases.



**Figure 4.7** Amplitude spectra of FM signals with sinusoidal modulating signal and fixed peak frequency deviation  $\Delta f$ .

## Bandwidth of Angle-Modulation Signals :

From equation (4-3), we observe that the spectrum consists of a carrier component at  $f_c$  plus an infinite number of sideband components at

$$f_c \pm n f_m \quad (n = 1, 2, \dots).$$

In fact, 98% of the normalized total signal power is contained in the bandwidth :

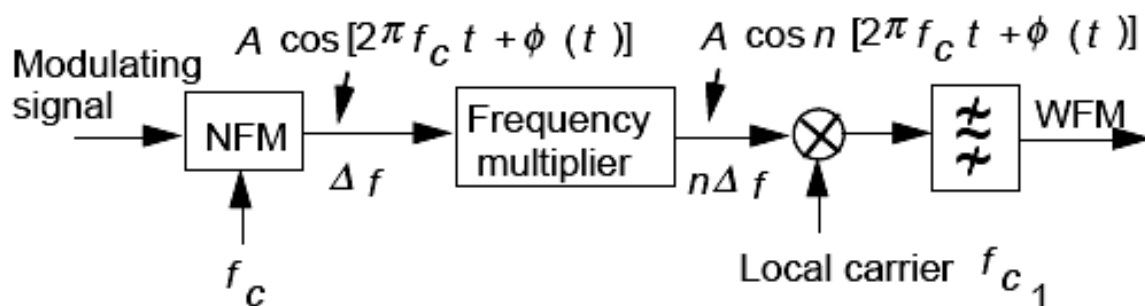
$$B_T = 2(\beta + 1) B$$

where  $\beta$  is either the phase modulation index or the frequency modulation index and B is the bandwidth of the modulating signal. The bandwidth of the angle-modulated signal with sinusoidal modulating signal depends on  $\beta$  and B. This is called **Carson's rule**. It gives a rule-of-thumb expression and an easy way to evaluate the transmission bandwidth of angle-modulated signals. When  $\beta \ll 1$ , the signal is a narrowband angle-modulated signal and its bandwidth is approximately equal to 2B.

## Generation of Wideband FM :

### 1-Indirect Method :

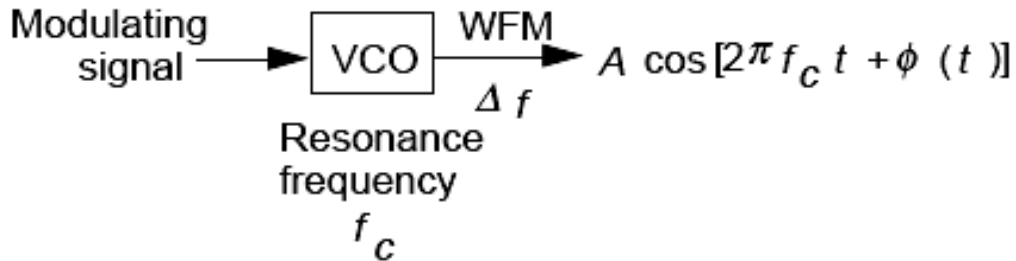
In this method, a narrowband frequency-modulated signal is first generated using an integrator and a phase modulator. A frequency multiplier is then used to increase the peak frequency deviation from  $\Delta f$  to  $n\Delta f$ . Use of frequency multiplication normally increases the carrier frequency from  $f_c$  to  $n f_c$ . A mixer or double-sideband modulator is required to shift the spectrum down to the desired range for further frequency multiplication or transmission. This is shown in Figure 4.8 .



**Figure 4.8** Indirect method of generating WFM.

## 2-Direct Method :

Here the carrier frequency is directly varied in accordance with the modulating signal. A common method used for generating direct FM is to vary the inductance  $L$  or capacitance  $C$  of a **voltage-controlled oscillator (VCO)**. This is shown in Figure 4.9 .



**Figure 4.9** Direct method of generating WFM.

The oscillator uses a high- $Q$  resonant circuit. Variations in the inductance or capacitance of the oscillator will change its oscillating frequency. Assuming that the capacitance of the tuned circuit varies linearly with the modulating signal  $m(t)$ , we have :

$$C = k m(t) + C_0$$

$$C = \Delta C + C_0$$

Where ,

$$\Delta C = k m(t)$$

$k$  is a constant and  $C_0$  is the capacitance of the VCO when the input signal to the oscillator is zero. The instantaneous frequency is given by :

$$f_i = \frac{1}{2\pi\sqrt{LC}}$$

$$f_i = \frac{1}{2\pi\sqrt{LC_0}\sqrt{1 + \frac{\Delta C}{C_0}}}$$

$$f_i = f_c \left[ 1 + \frac{\Delta C}{2C_0} \right]^{-\frac{1}{2}}$$

where the zero-input-signal resonance frequency is :

$$f_c = \frac{1}{2\pi\sqrt{LC_0}}$$

For  $\Delta C \ll C_0$ , we can write :

$$f_i \approx f_c \left[ 1 - \frac{\Delta C}{2C_0} \right]$$

$$f_i \approx f_c \left[ 1 - \frac{km(t)}{2C_0} \right]$$

$$f_i \approx f_c - \Delta f$$

where ,

$$\Delta f = \frac{km(t)}{2C_0} f_c = \frac{\Delta C}{2C_0} f_c$$

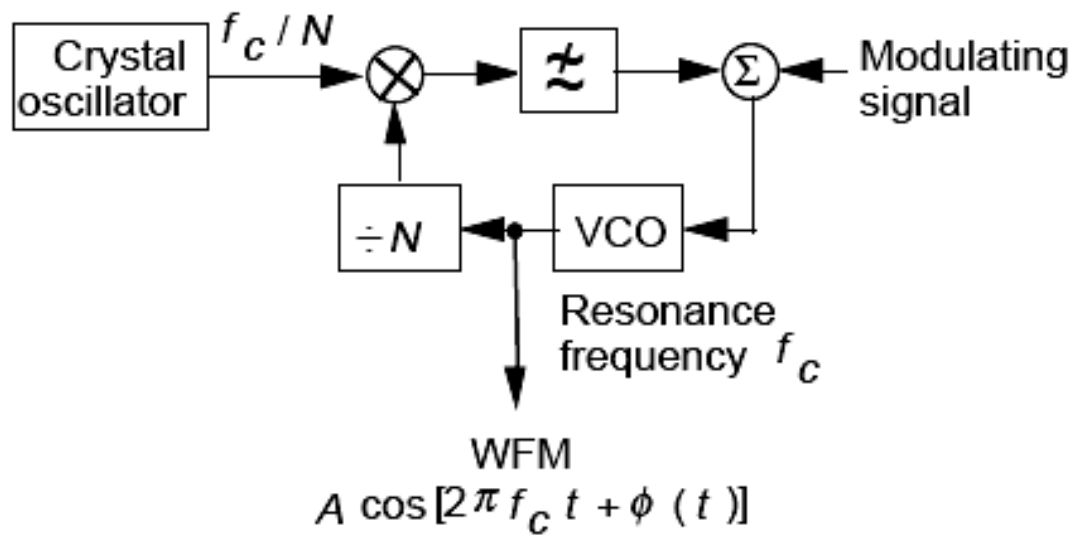
Although the change in capacitance may be small, the frequency deviation  $\Delta f$  may be quite large if the resonance frequency  $f_c$  is large. We can alternatively vary the inductance to achieve the same effect.

**Advantage** - Large frequency deviations are possible and thus less frequency multiplication is needed.

**Disadvantage** - The carrier frequency tends to drift and additional circuitry is required for frequency stabilization.

To stabilize the carrier frequency, a phase-locked loop can be used. This is shown in Figure 4.10 .





**Figure 4.10** Direct method of generating WFM with frequency stabilization.

## Angle Demodulation

### Review of Angle Modulation :

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM).

We have seen that an angle-modulated signal can be represented by :

$$s_c(t) = A \cos \theta(t)$$

$$s_c(t) = A \cos [2\pi f_c t + \phi(t)]$$

where  $A$  is a constant,

$$\theta(t) = 2\pi f_c t + \phi(t),$$

and  $f_c$  is the carrier frequency.  $\phi(t)$  is a function of the modulating signal  $m(t)$  and is given by :

$$\phi(t) = \begin{cases} k_p m(t) \dots \dots \dots \text{for..PM} \\ k_f \int_{-\infty}^t m(\lambda) d\lambda \dots \dots \dots \text{for..FM} \end{cases}$$

and

$$\frac{d\phi(t)}{dt} = \begin{cases} k_p \frac{dm(t)}{dt} \dots \dots \dots \text{for..PM} \\ k_f m(t) \dots \dots \dots \text{for..FM} \end{cases}$$

The instantaneous angular frequency of  $s_c(t)$  is defined as :

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

In terms of frequency, the instantaneous frequency of  $s_c(t)$  is :

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$f_i(t) = \begin{cases} f_c + \frac{1}{2\pi} k_p \frac{dm(t)}{dt} \dots \dots \dots \text{for..PM} \\ f_c + \frac{1}{2\pi} k_f m(t) \dots \dots \dots \text{for..FM} \end{cases}$$

$\frac{1}{2\pi} \frac{d\phi(t)}{dt}$  is known as the instantaneous frequency deviation.

## Angle Demodulation :

Demodulation of an angle-modulated signal requires a circuit that produces an output proportional to the instantaneous frequency  $f_i(t)$  or the instantaneous

frequency deviation  $\frac{1}{2\pi} \frac{d\phi(t)}{dt}$  of the input signal to the demodulator.

## Frequency Discrimination :

Frequency discrimination is a frequency-to-amplitude conversion process. Consider an angle-modulated signal :

$$s_c(t) = A \cos \theta(t) \quad \dots\dots\dots(4-4)$$

where ,

$$\theta(t) = 2\pi f_c t + \phi(t) \quad \dots\dots\dots(4-5)$$

$$\phi(t) = \begin{cases} k_p m(t) \dots\dots\dots \text{for..PM} \\ k_f \int_{-\infty}^t m(\lambda) d\lambda \dots\dots\dots \text{for..FM} \end{cases} \quad \dots\dots\dots(4-6)$$

and

$$\frac{d\phi(t)}{dt} = \begin{cases} k_p \frac{dm(t)}{dt} \dots\dots\dots \text{for..PM} \\ k_f m(t) \dots\dots\dots \text{for..FM} \end{cases} \quad \dots\dots\dots(4-7)$$

If we differentiate equation (4-4), we get :

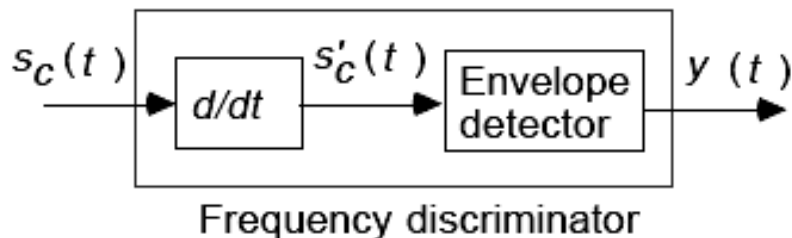
$$\begin{aligned} s'(t) &= -A \sin \theta(t) \frac{d\theta(t)}{dt} \\ &= -A \sin \theta(t) \left[ 2\pi f_c + \frac{d\phi(t)}{dt} \right] \\ &= -A \left[ 2\pi f_c + \frac{d\phi(t)}{dt} \right] \sin [2\pi f_c t + \phi(t)] \end{aligned}$$

The signal is both amplitude- and angle-modulated. If we pass the signal to an envelope detector, we get :

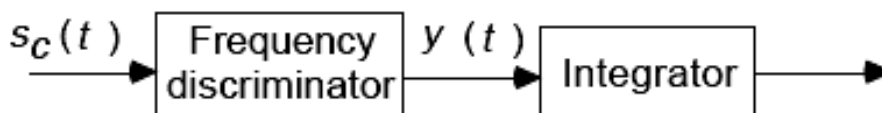
$$y(t) = A \left[ 2\pi f_c + \frac{d\phi(t)}{dt} \right]$$

$$y(t) = \begin{cases} A \left[ 2\pi f_c + k_p \frac{dm(t)}{dt} \right] \dots\dots \text{for..PM} \\ A \left[ 2\pi f_c + k_f m(t) \right] \dots\dots\dots \text{for..FM} \end{cases}$$

Knowing the values of  $A$ ,  $f_c$ ,  $k_p$  and  $k_f$ , we can compute the desired signal  $m(t)$  from  $y(t)$ . Figure 4.11 shows the circuit for frequency demodulation. The differentiator followed by an envelope detector is called a **frequency discriminator**. For demodulation of PM signals, we simply integrate the output of a frequency discriminator. This yields a signal which is proportional to  $m(t)$ . Figure 4.12 shows the circuit for phase demodulation.



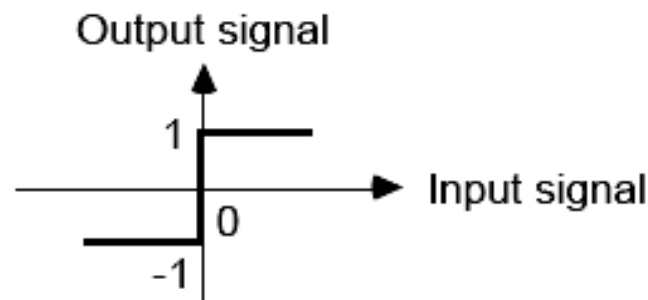
**Figure 4.11** Frequency demodulation using a frequency discriminator.



**Figure 4.12** Phase demodulation using a frequency discriminator and an integrator.

In practice, channel noise and other factors may cause  $A$  to vary. If  $A$  varies,  $y(t)$  will vary with  $A$ . Hence, it is essential to maintain the amplitude of the input signal to the frequency discriminator. A **hard limiter** is usually used to eliminate any amplitude variations. A hard limiter is a device which limits the output signal to (say) +1 or -1 volt.

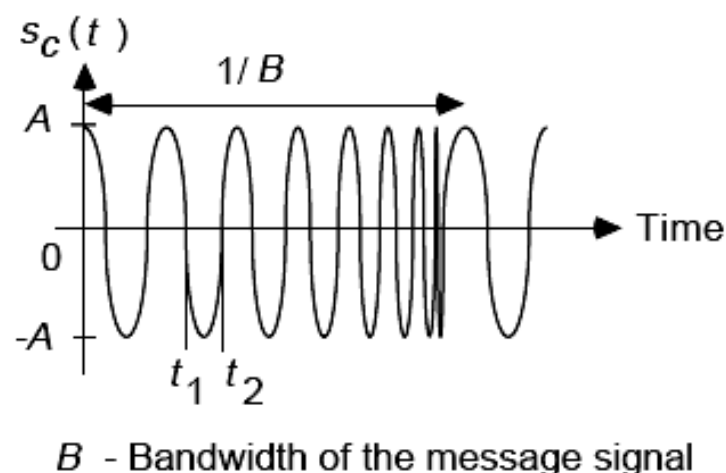
Figure 4.13 shows the input-output characteristic of a hard limiter.



**Figure 4.13** Input-output characteristic of a hard-limiter.

### Zero-Crossing Detection:

We have seen that a hard limiter is usually used to eliminate any amplitude fluctuation. The message signal must therefore be contained in the points where the angle-modulated signal crosses the zero voltage level. This produces a means of demodulating an angle-modulated signal. Consider the angle-modulated signal as shown in Figure 4.14 .



**Figure 4.14** Zero-crossing determination.

Let  $t_1$  and  $t_2$  be two adjacent zero-crossing points, where  $t_2 > t_1$ . Integrating  $\theta(t)$ , we have :

$$\int_{t_1}^{t_2} d\theta(t) = \int_{t_1}^{t_2} 2\pi f_i(t) dt$$

$$\theta(t_2) - \theta(t_1) = \begin{cases} \int_{t_1}^{t_2} \left[ 2\pi f_c + k_p \frac{dm(t)}{dt} \right] dt \dots \dots \dots \text{for..PM} \\ \int_{t_1}^{t_2} \left[ 2\pi f_c + k_f m(t) \right] dt \dots \dots \dots \text{for..FM} \end{cases}$$

but also ,

$$\theta(t_2) - \theta(t_1) = \pi$$

For  $f_c \gg B$  (the bandwidth of the message signal),  $dm(t)/dt$  for PM signals and  $m(t)$  for FM signals change much more slowly than  $f_c$ .  $dm(t)/dt$  and  $m(t)$  may be assumed constant in the interval  $t_2 - t_1$ .

We can write :

$$\pi = \begin{cases} \left[ 2\pi f_c + k_p \frac{dm(t)}{dt} \right] (t_2 - t_1) \dots \dots \dots \text{for..PM} \\ \left[ 2\pi f_c + k_f m(t) \right] (t_2 - t_1) \dots \dots \dots \text{for..FM} \end{cases}$$

$$\pi = 2\pi f_i(t) [t_2 - t_1]$$

$$f_i(t) = \frac{1}{2(t_2 - t_1)}$$

where ,

$$f_i(t) = \begin{cases} f_c + \frac{1}{2\pi} k_p \frac{dm(t)}{dt} \dots\dots\dots \text{for..PM} \\ f_c + \frac{1}{2\pi} k_f m(t) \dots\dots\dots \text{for..FM} \end{cases}$$

Knowing the values of  $f_c$ ,  $k_p$  and  $k_f$ , the desired signal  $m(t)$  may be found by measuring the spacing between zero crossings in the interval  $t_2 - t_1$ . A detector utilizing this technique is called a **zero-crossing detector**. For demodulation of PM signals, we simply integrate the output of a zero-crossing detector. Again, this yields a signal which is proportional to  $m(t)$ .

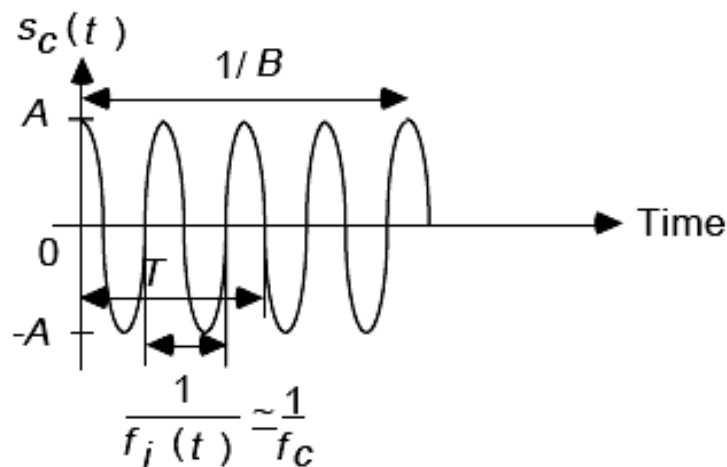
In practice, we consider counting  $n$  number of zero-crossings in a time interval  $T$ , where :

$$\frac{1}{f_c} \ll T \ll \frac{1}{B}$$

and  $B$  is the bandwidth of the message signal. This is shown in Figure 4.15. Then, the number of zero crossings in a time interval  $T$  is :

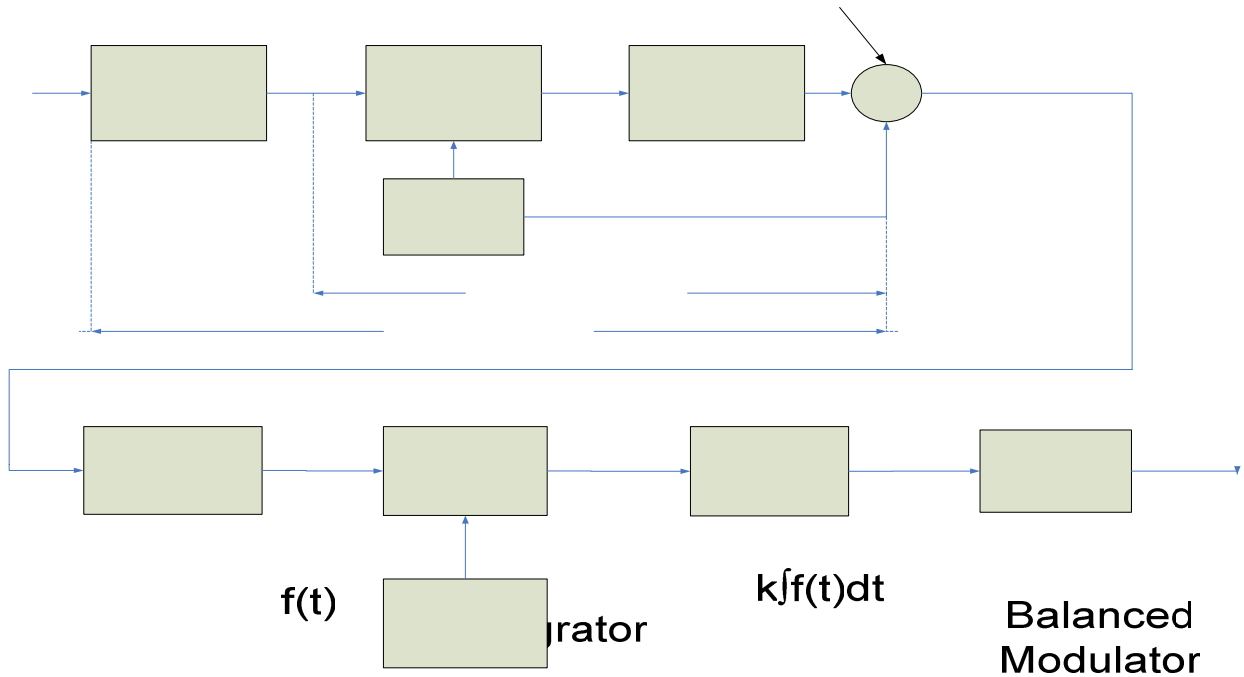
$$n = \frac{T}{2(t_2 - t_1)}$$

$$\frac{n}{T} = f_i(t)$$



**Figure 4.15** Counting intervals.

Armstrong-type indirect FM transmitter:



The transmitter shown above operates at an unmodulated carrier frequency of 91.2MHz with a maximum-frequency deviation of 75 kHz .

The transmitter is crystal-controlled , starting at the controlled carrier frequency of 200kHz .

There is a total frequency multiplication of  $64 \times 48 = 3072$ , so that the maximum modulation index at the multiplier input can be fixed at 0.5 radian.

An injected frequency of 10.9 MHz is used to shift the carrier frequency down from 12.8 to 1.9 MHz between multiplier sections.

Narrowband FM

Frequency Multiplier X 64

$f_c = 12.8 \text{ MHz}$   
 $\Delta f = 1.6 \text{ kHz}$   
 $m_f < 32 \text{ rad.}$

Frequency converter

$f_c =$   
 $\Delta f =$   
 $m_f <$